# Collaborative Delivery with Energy-Constrained Mobile Robots

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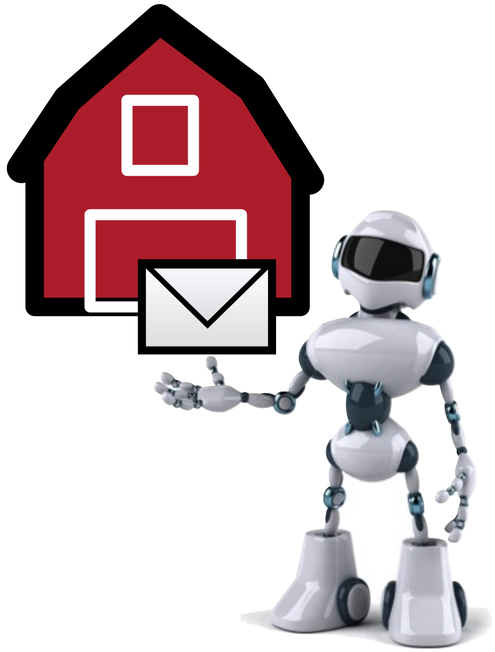

Image Sources: *BLADE Nano QX RTF Quadcopter*, http://paralebin.hol.es/robot/, http://www.generationrobots.com

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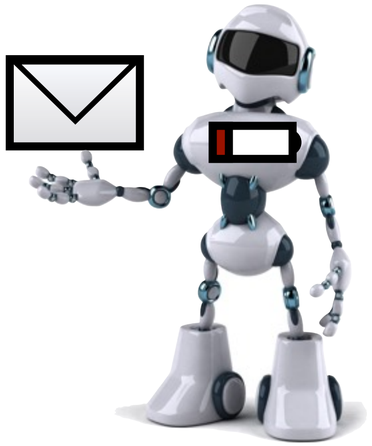

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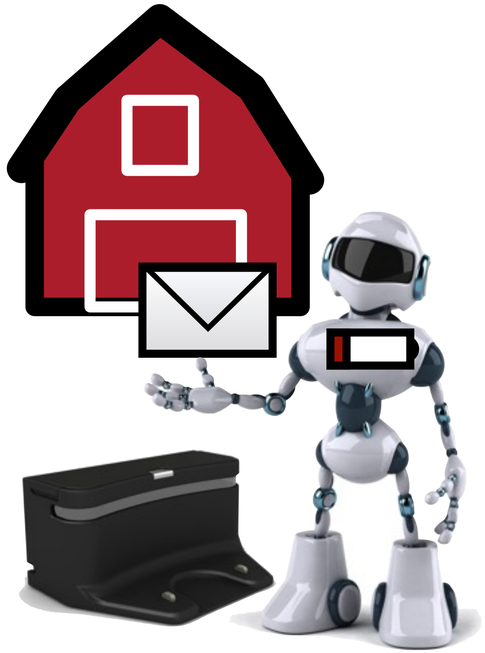

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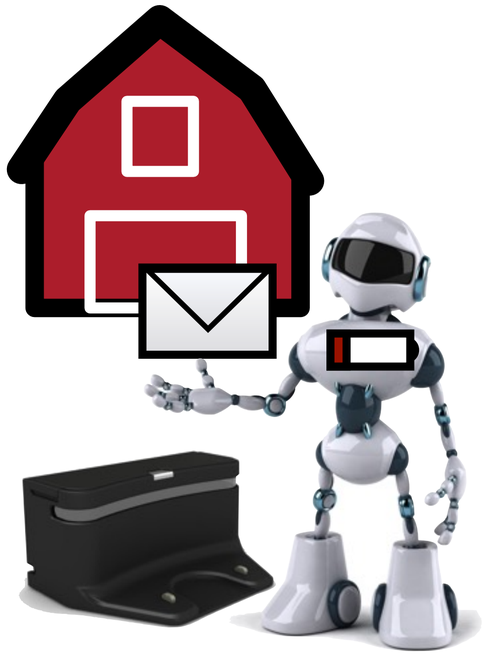

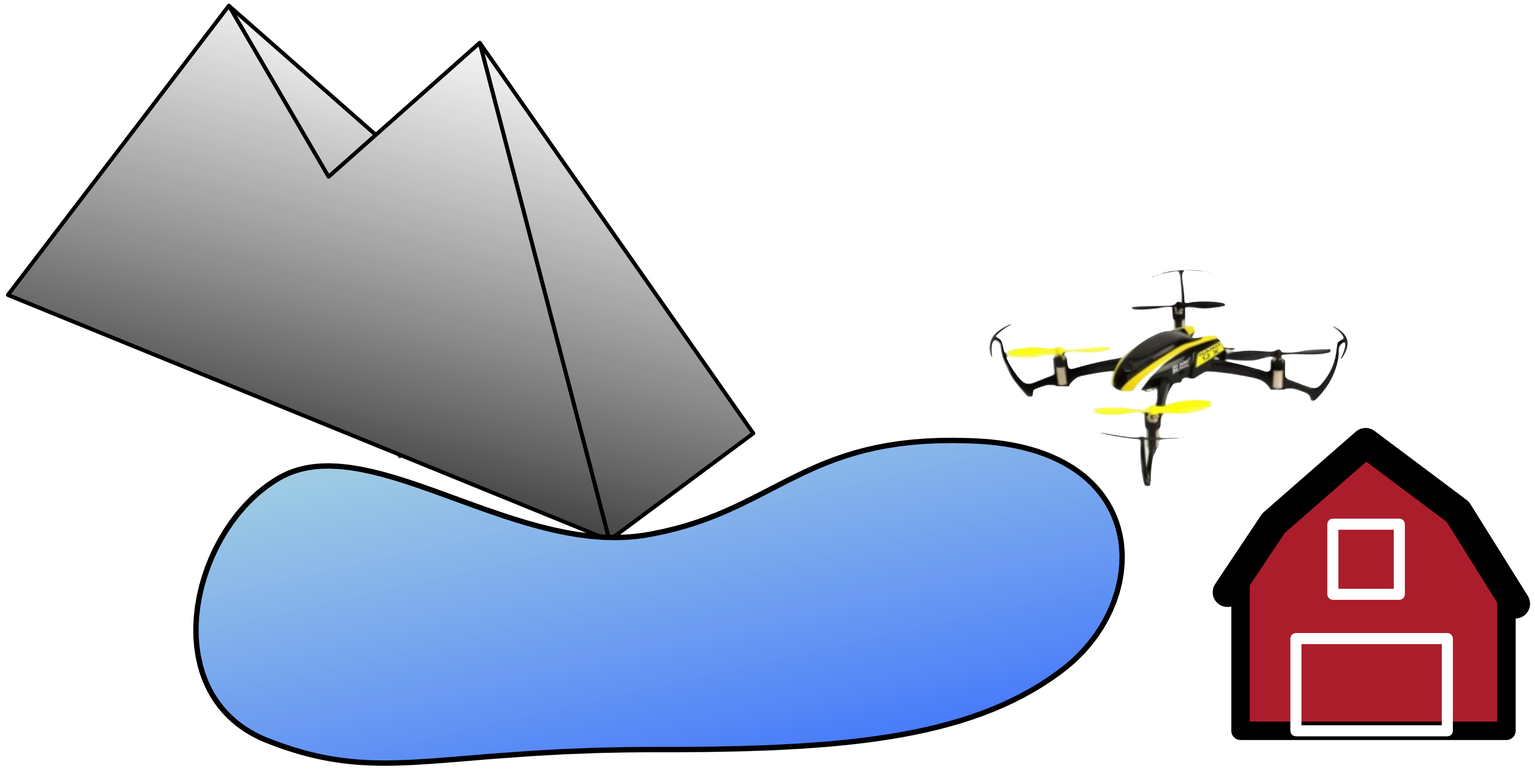

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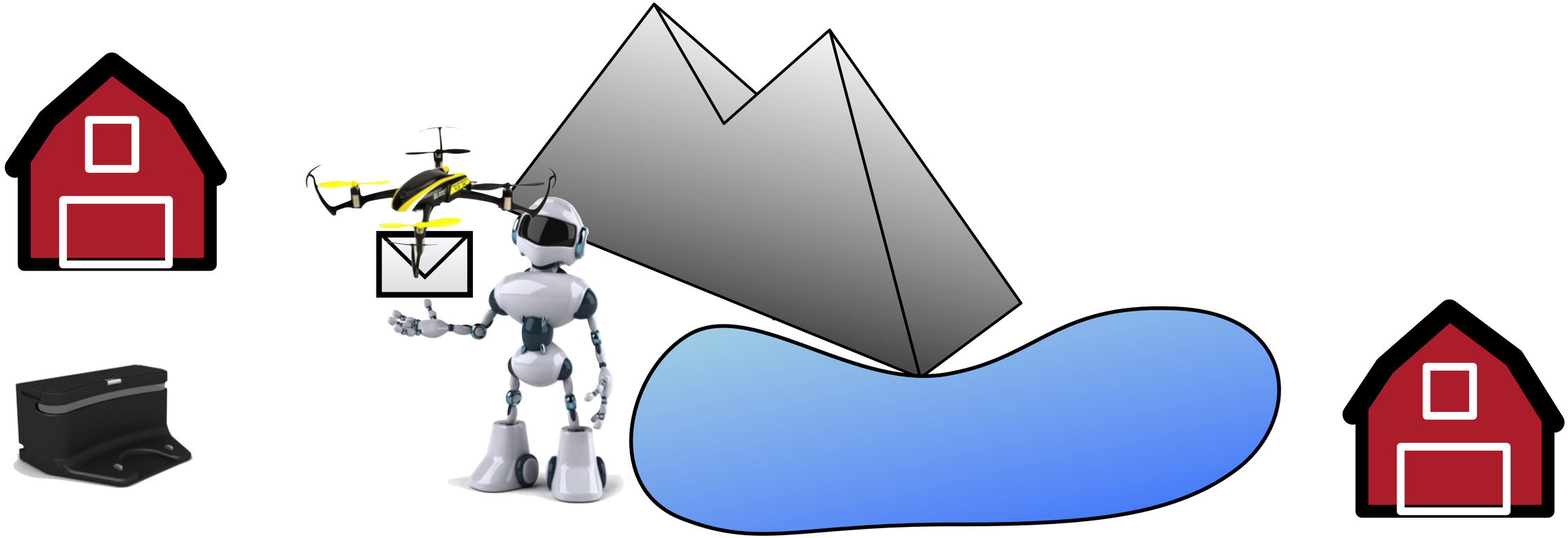

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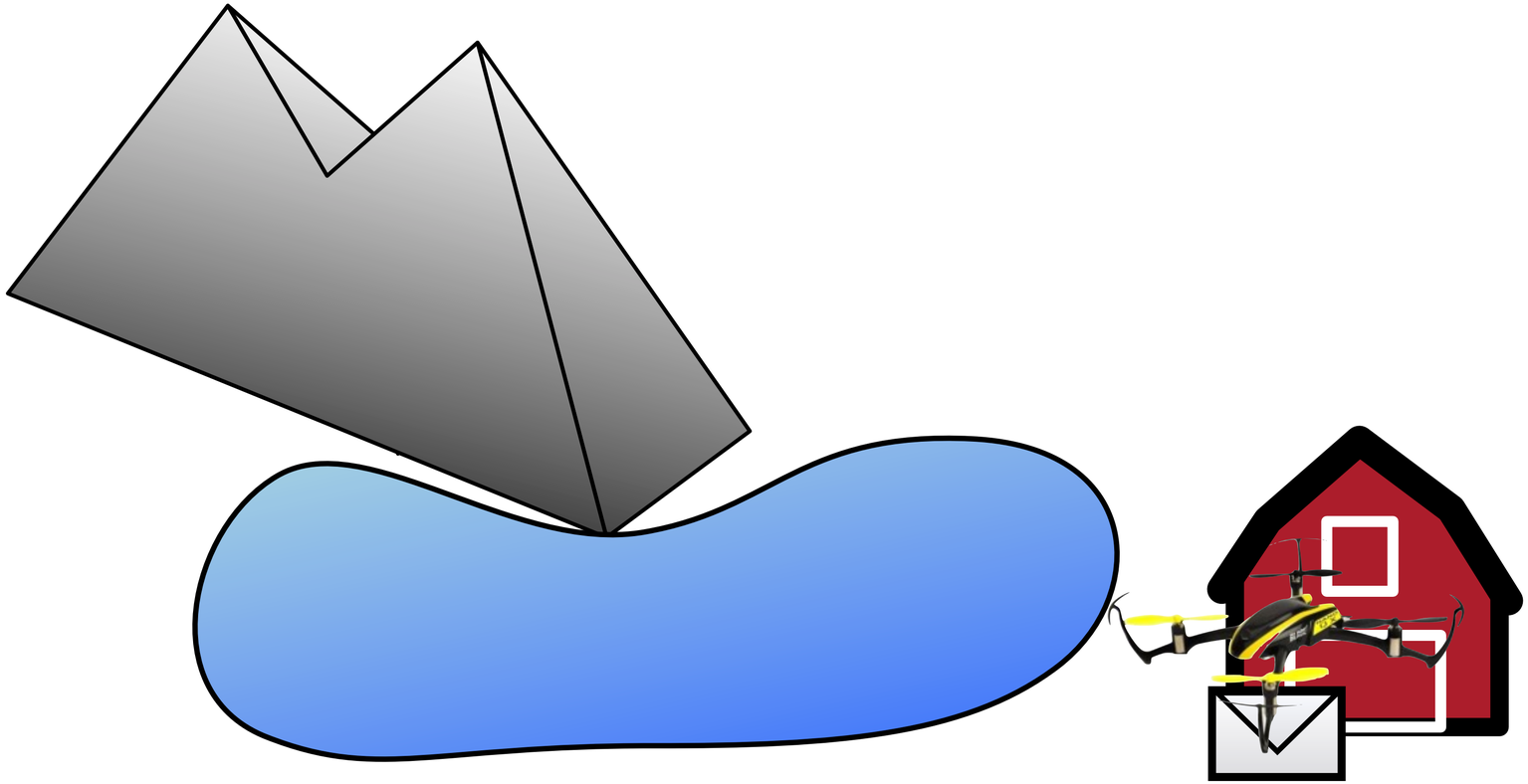

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## Setting

- undirected, edge-weighted graph with $n=|V|$, $m=|E|$
- single message from $s$ to $t$
- anyone can use any edge

## Assumptions

- global coordination
- handovers anywhere

## Agents

- $k$ agents each with
  - starting vertex $p_i$
  - battery capacity $B_i$
- agents have to return

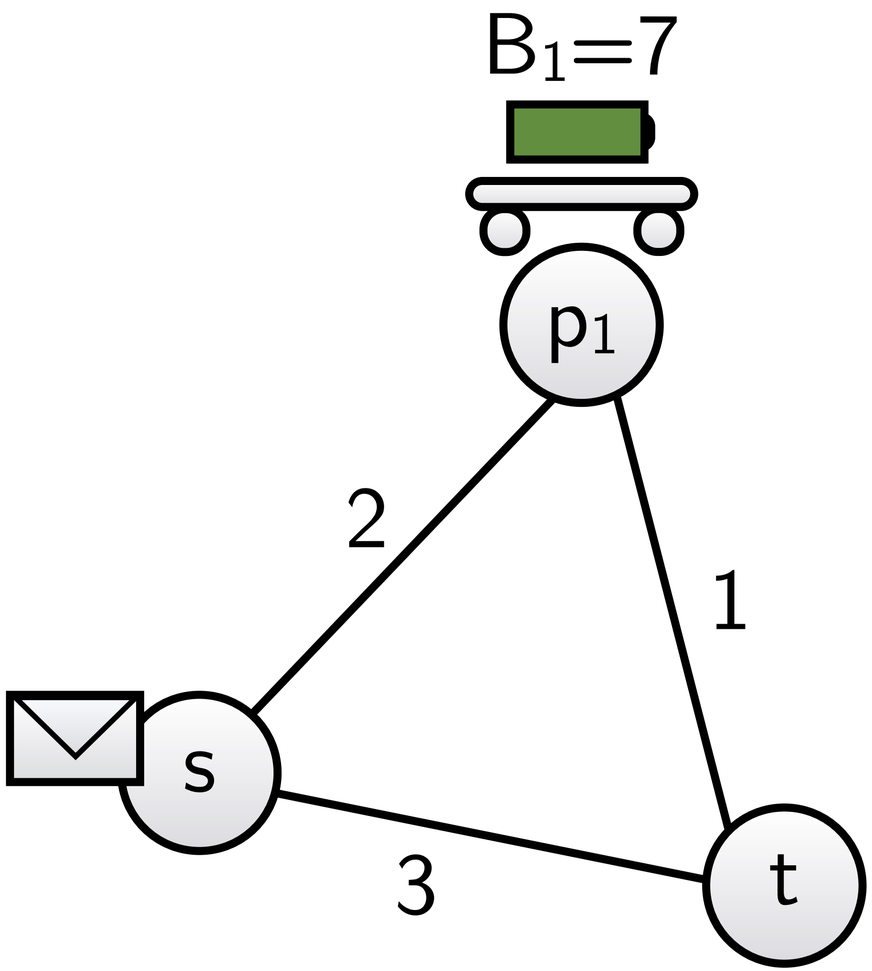
## Task

Decide feasibility!

If yes, compute a schedule.

# Example





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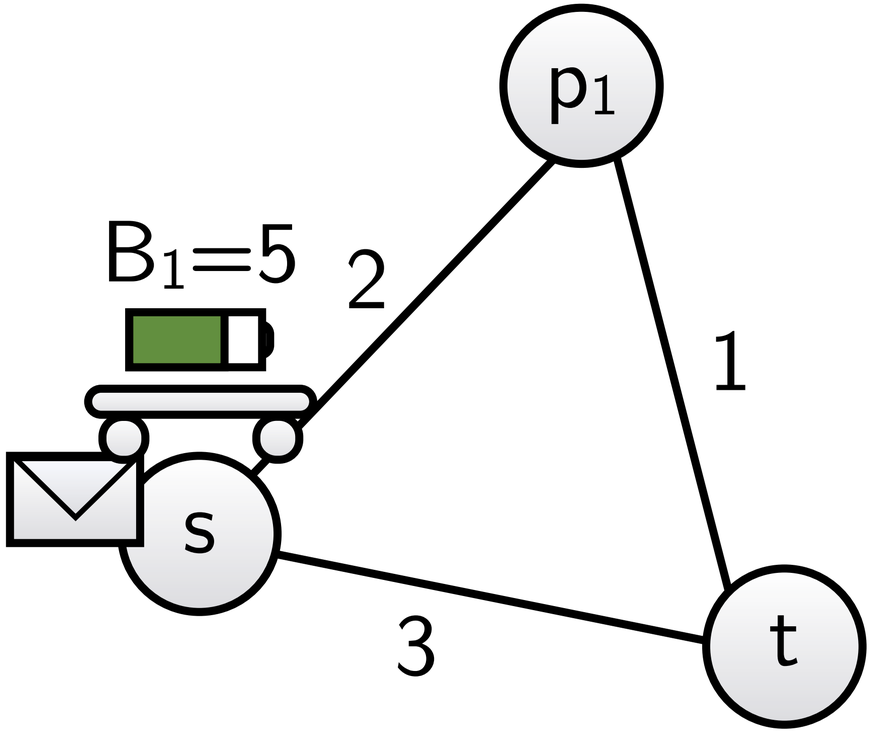

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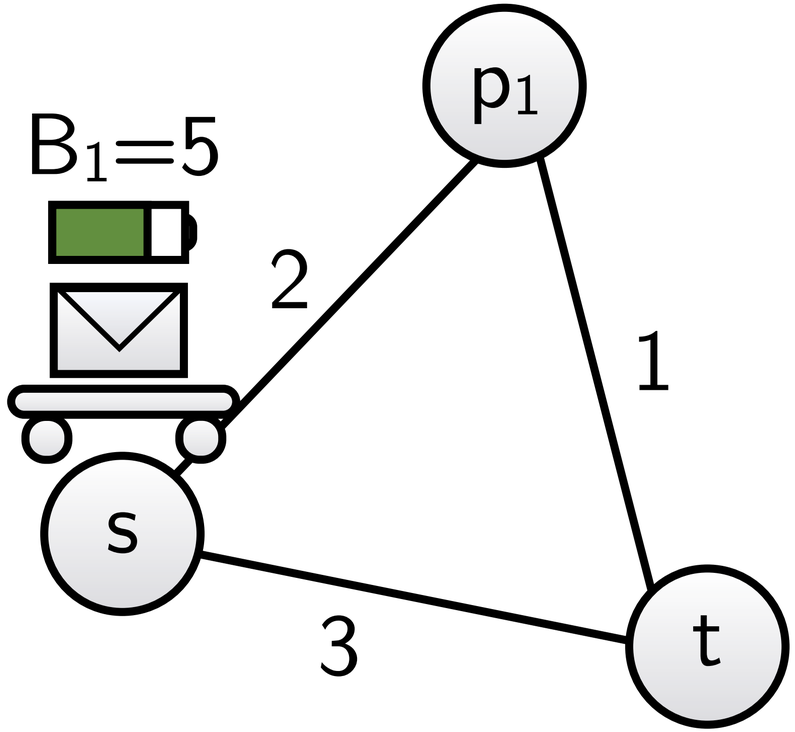

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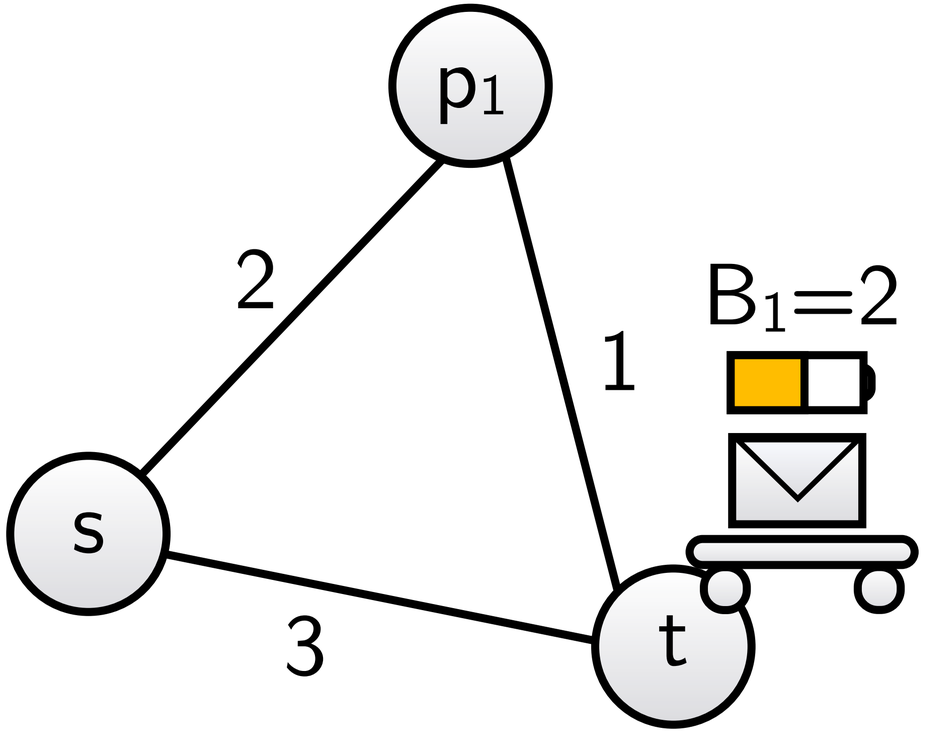

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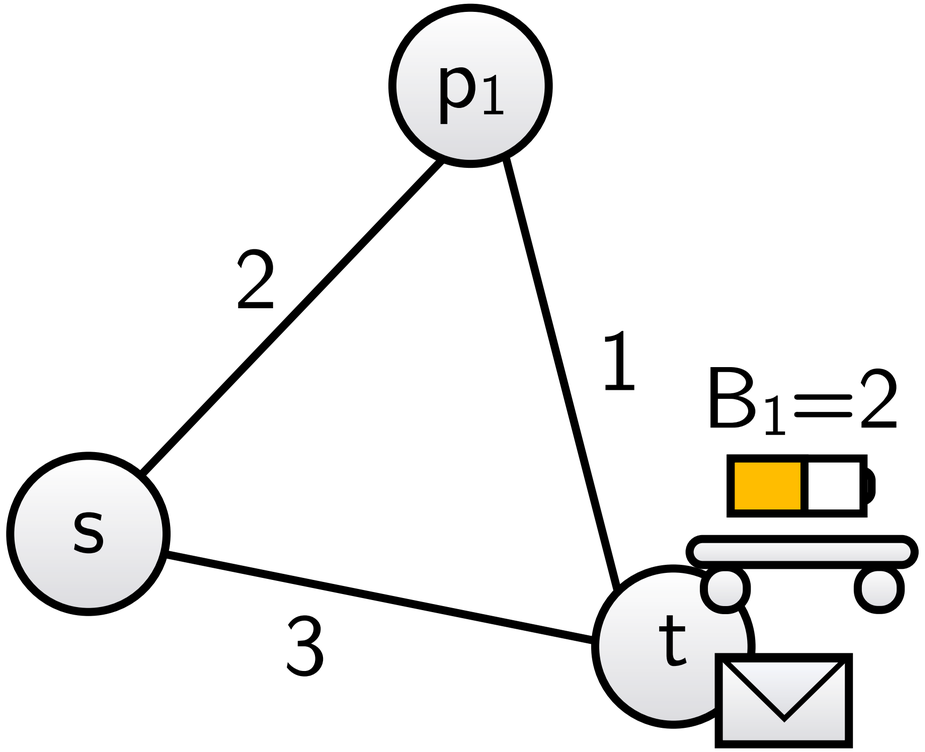

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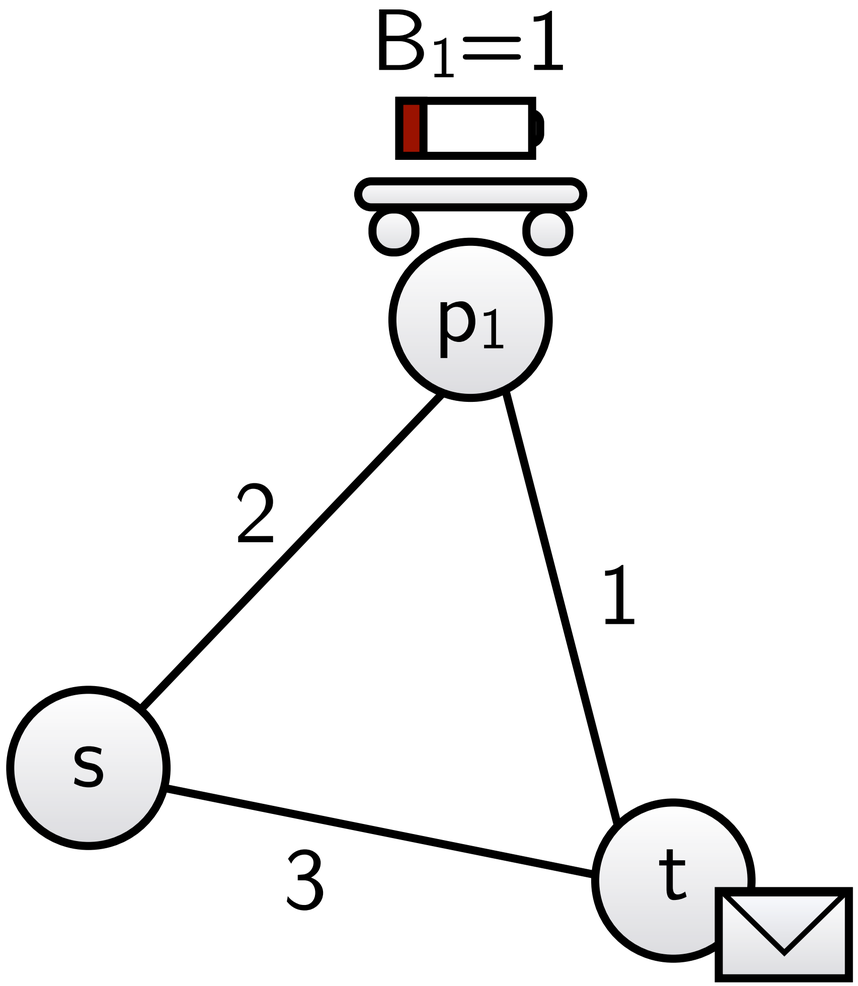

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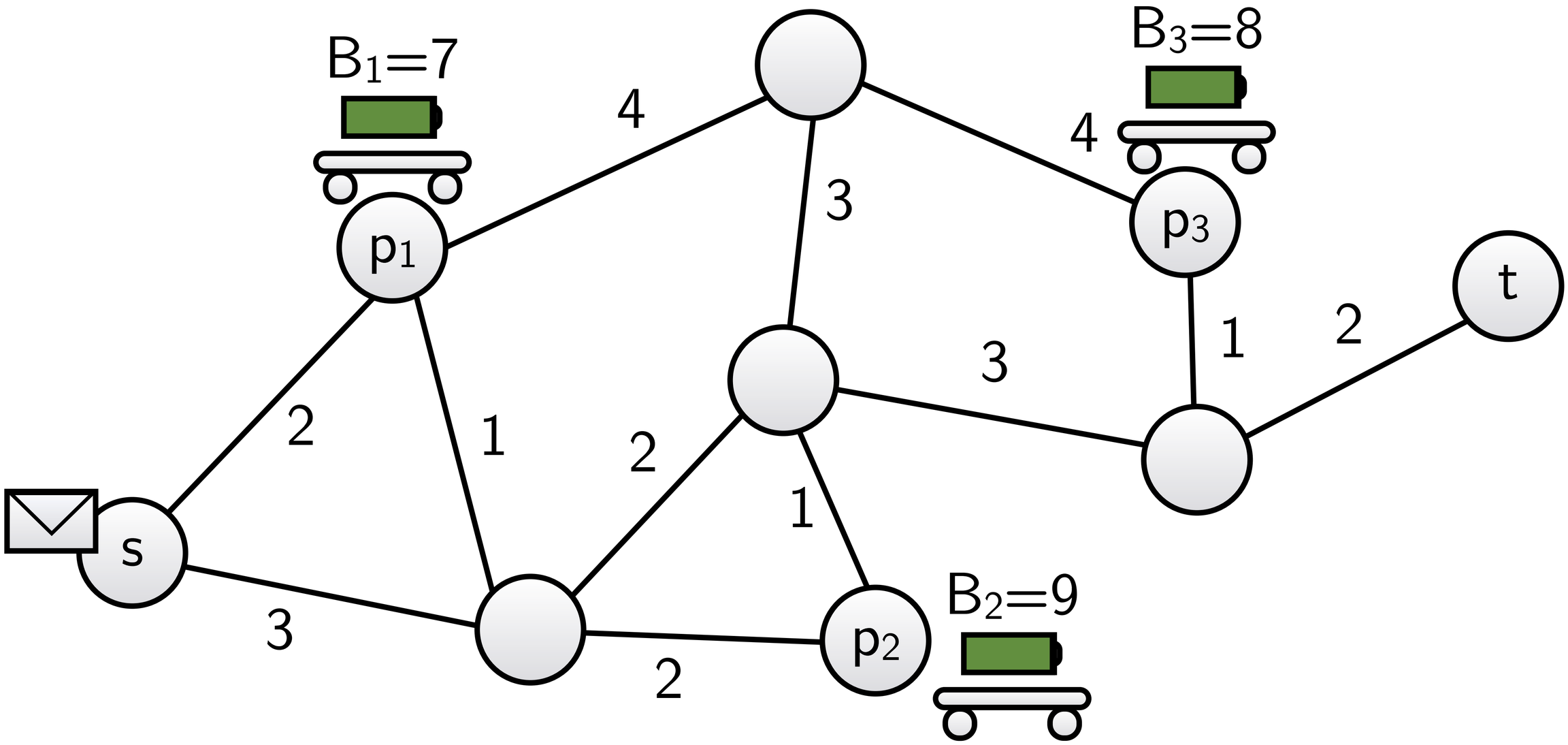

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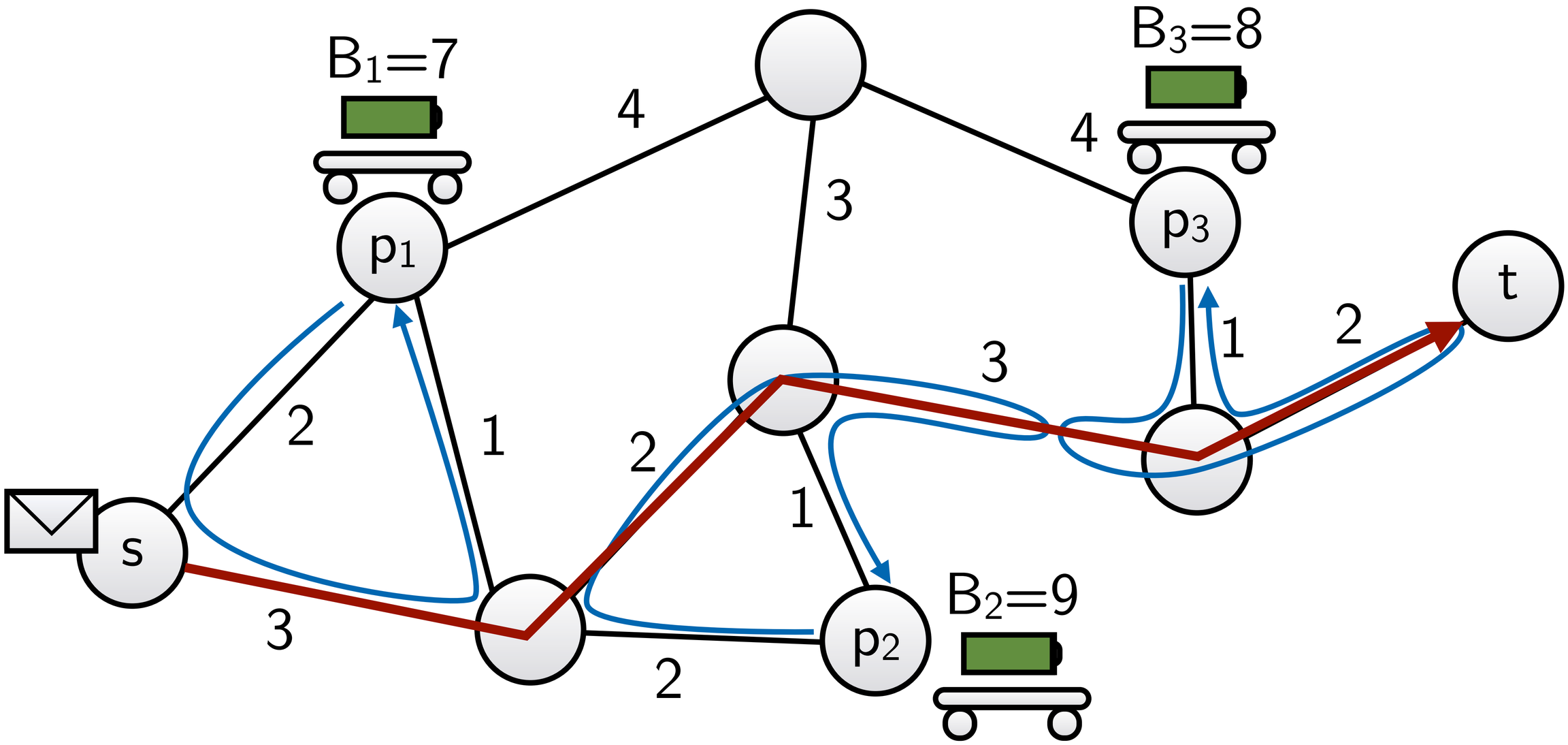
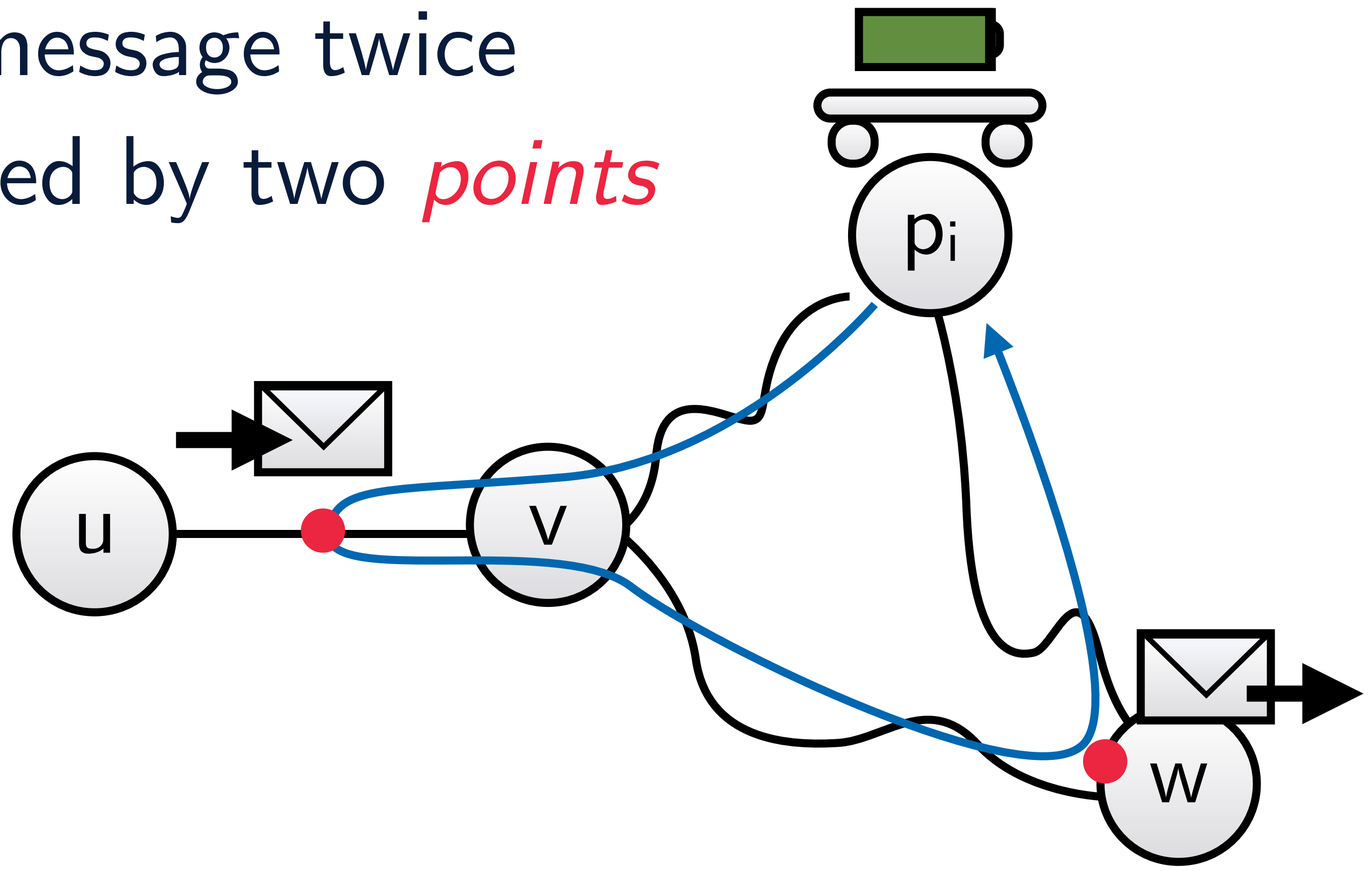


## Observations

- No agent picks up the message twice
- Path of an agent specified by two *points*



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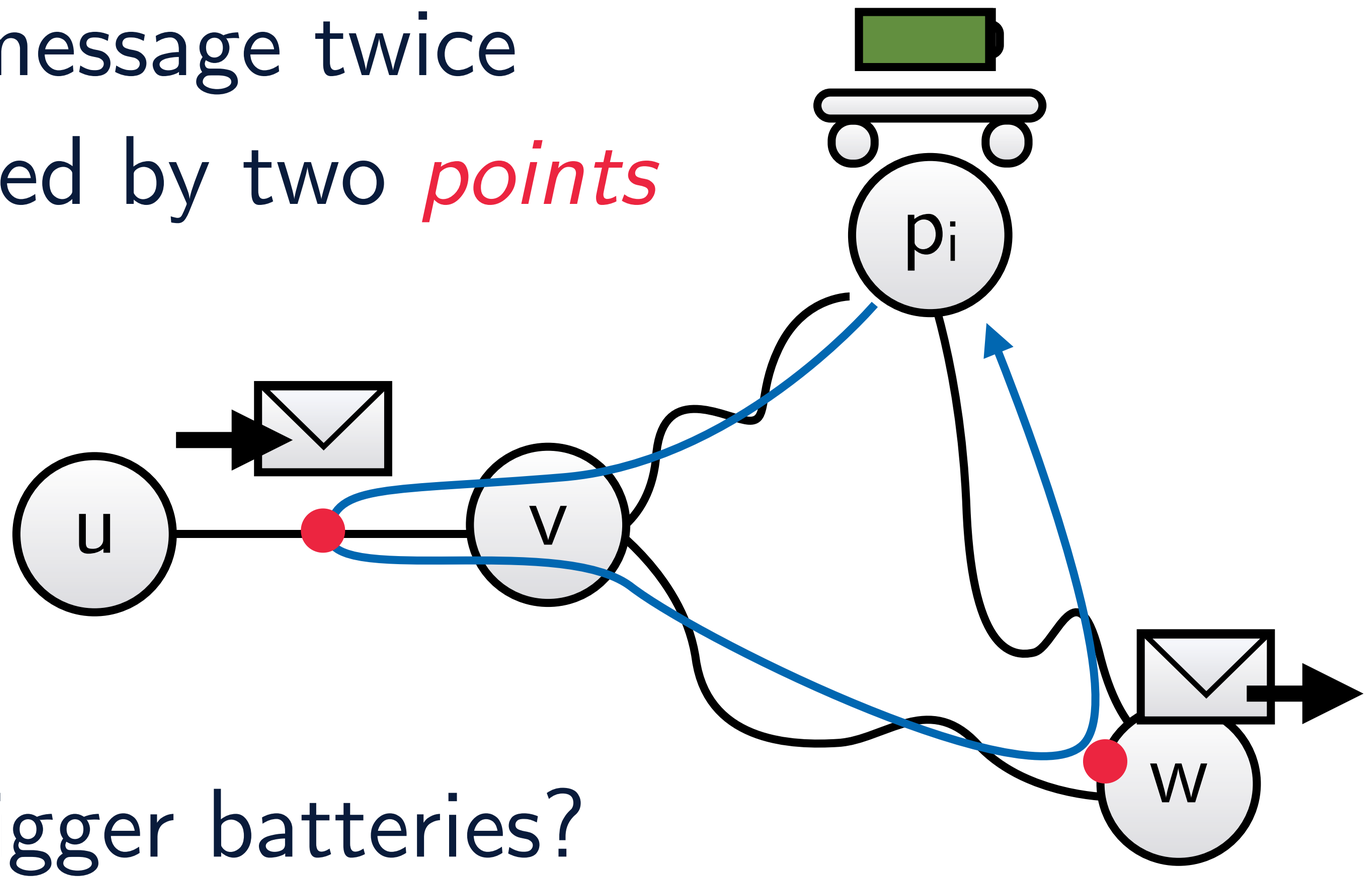
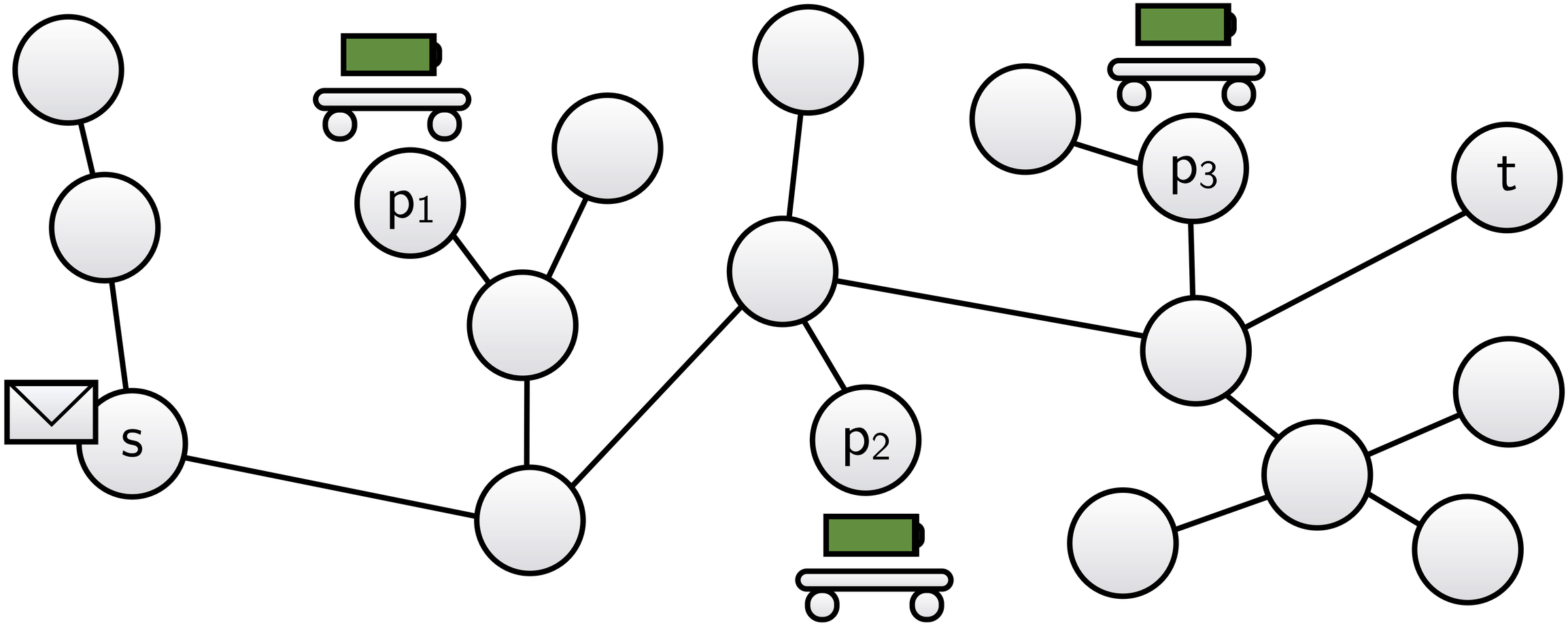
## Overview of the talk

- For trees?
- For all graphs?
- What can we do with bigger batteries?

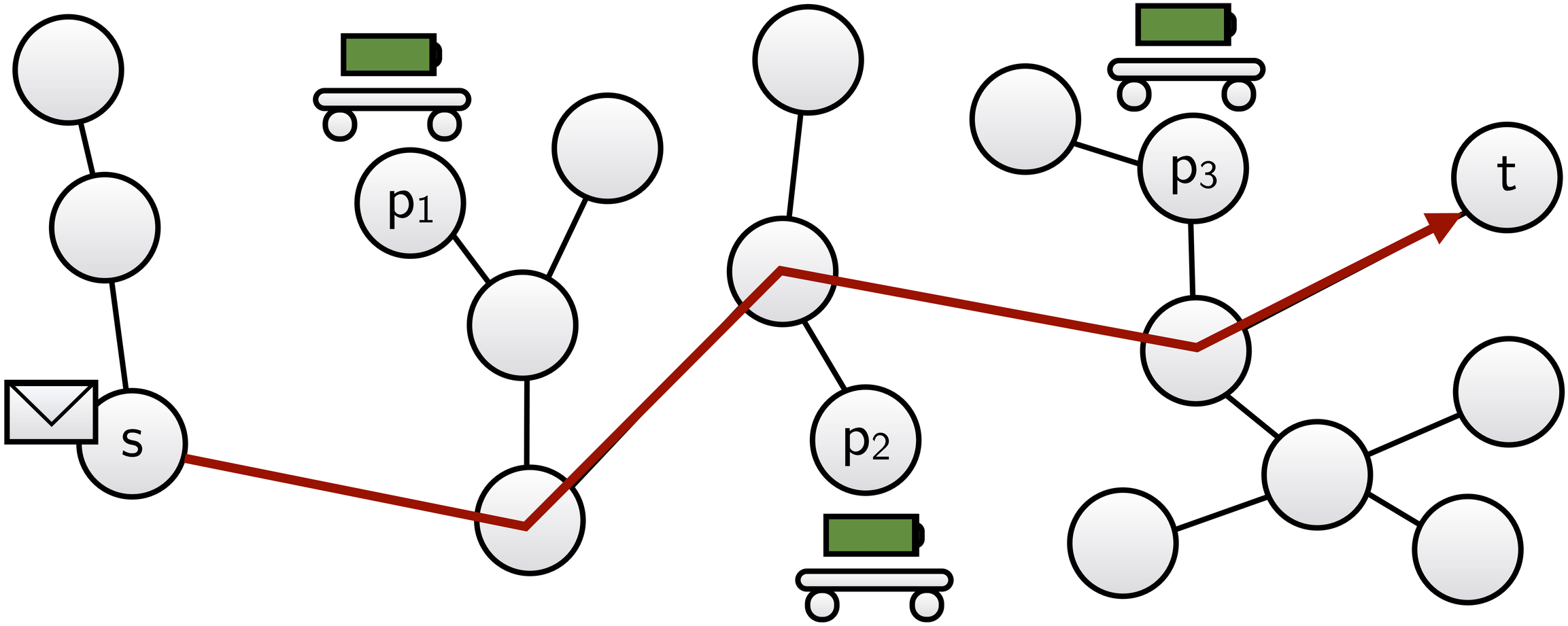


# Reduction to Paths





Move the agents to the $s$-$t$-path and adapt the budgets.



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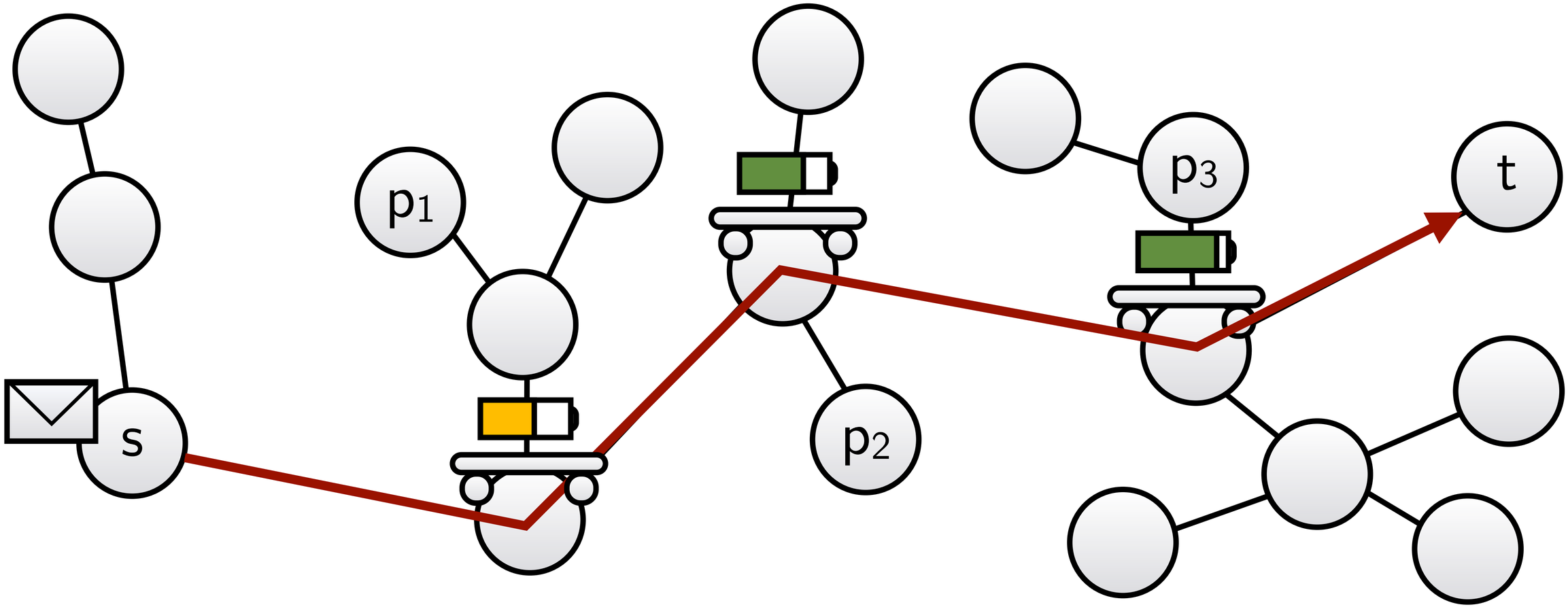

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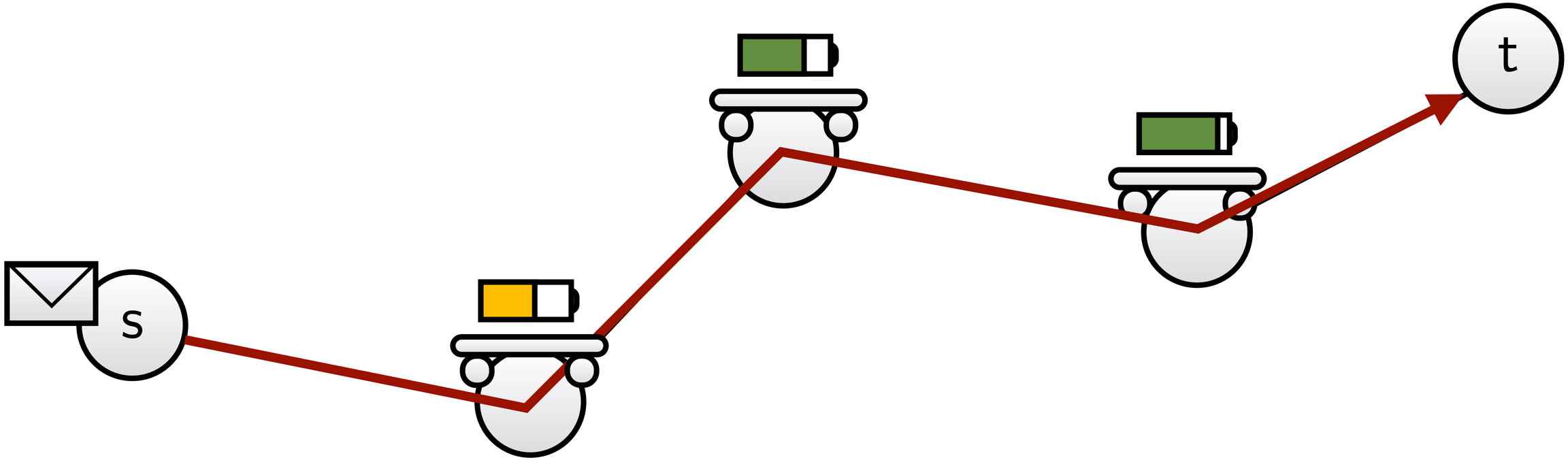

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interval covering problem

- length $B_i/2$ containing $p_i$

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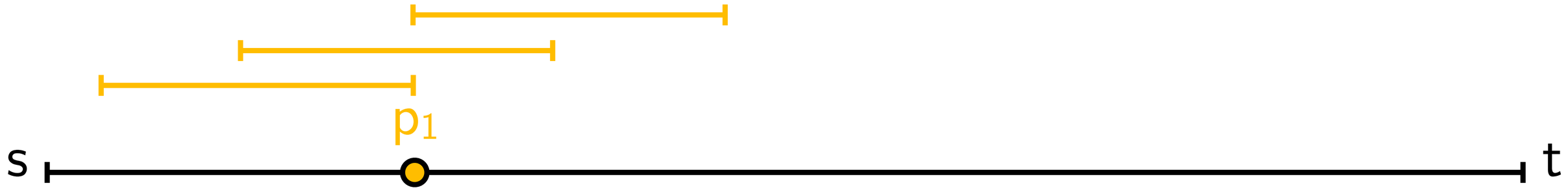

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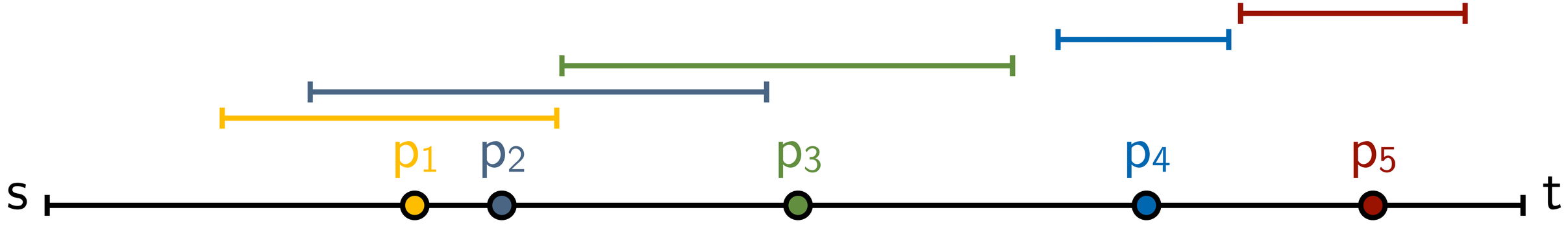

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## Greedy Algorithm

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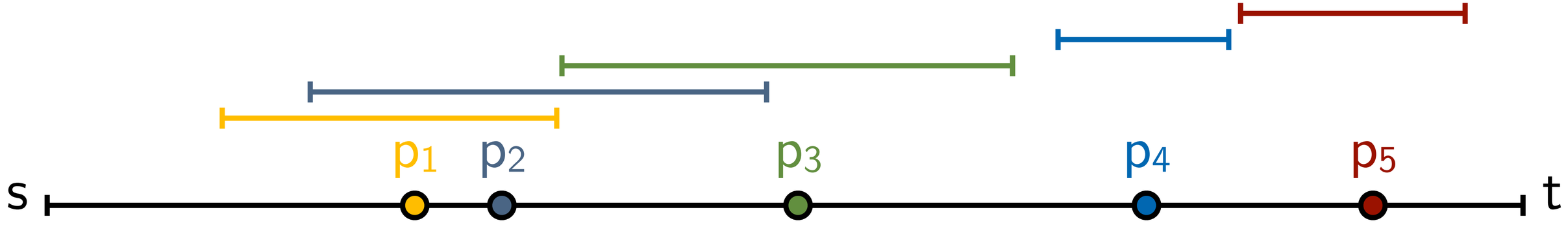

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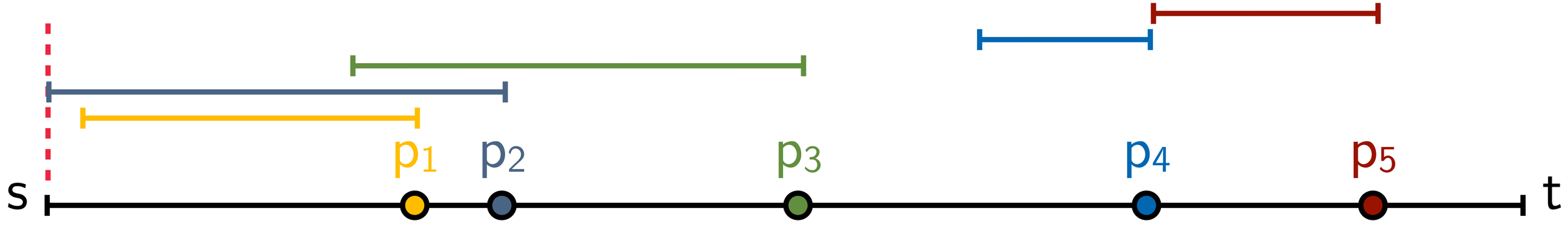

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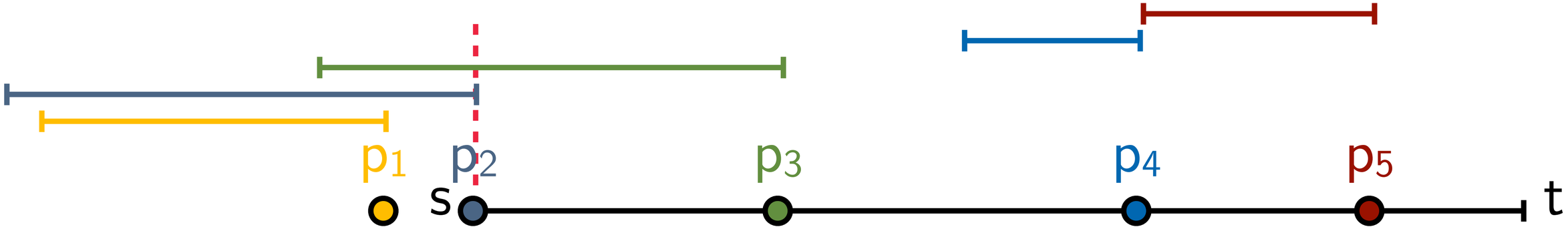

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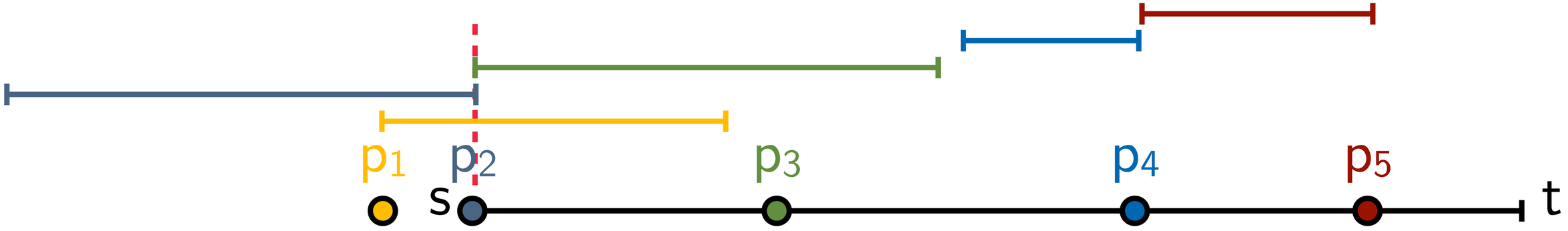

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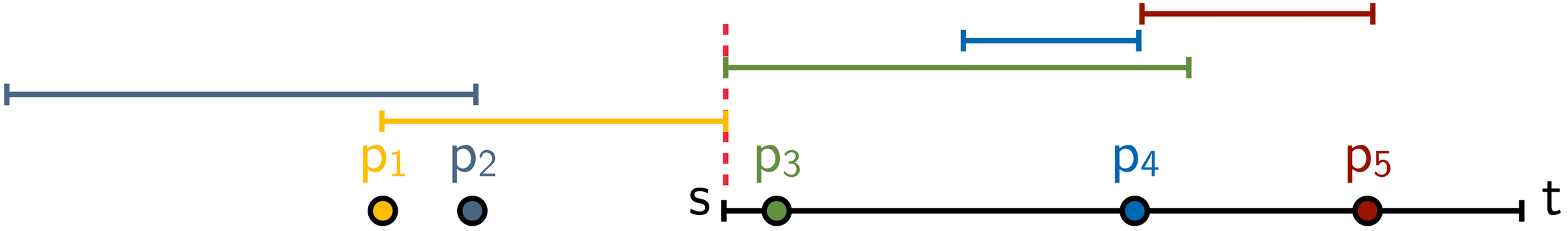

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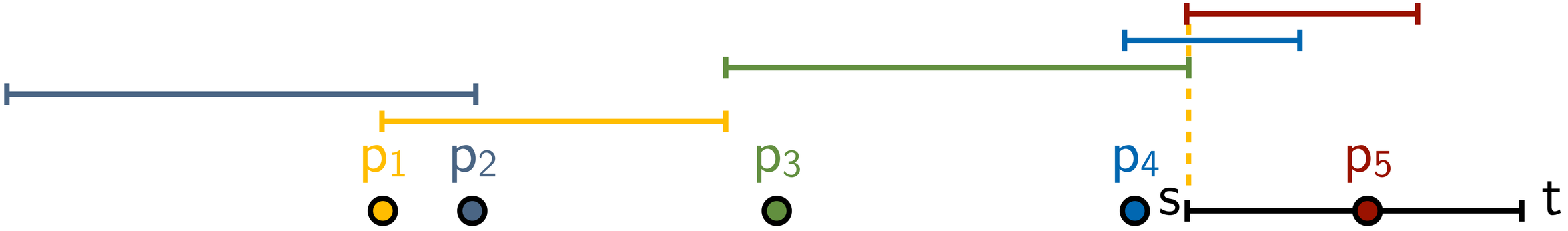

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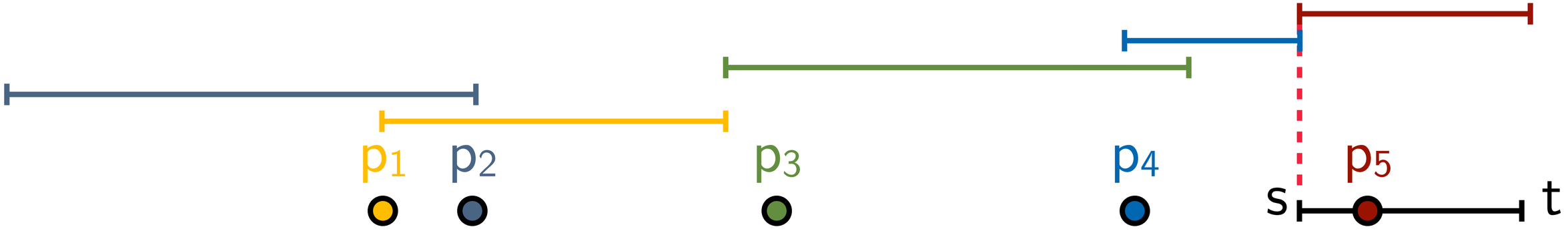

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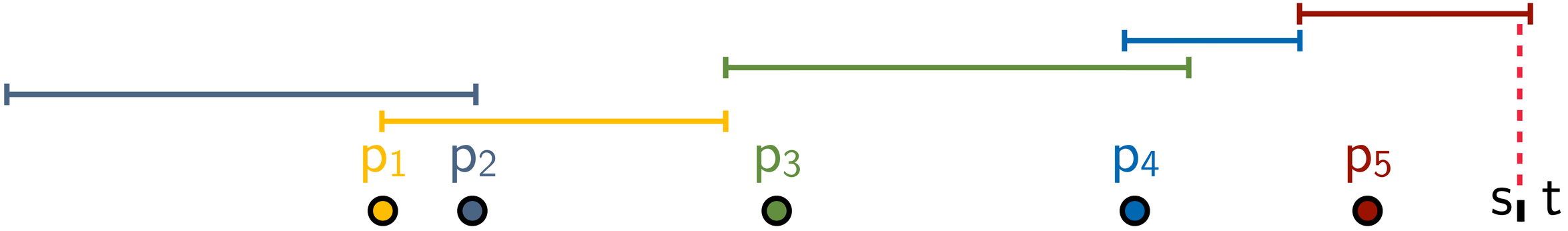

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**Proof:** exchange argument

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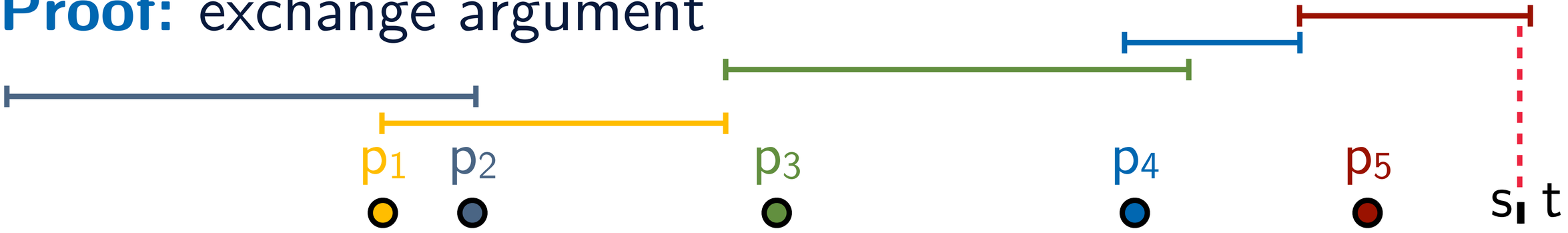
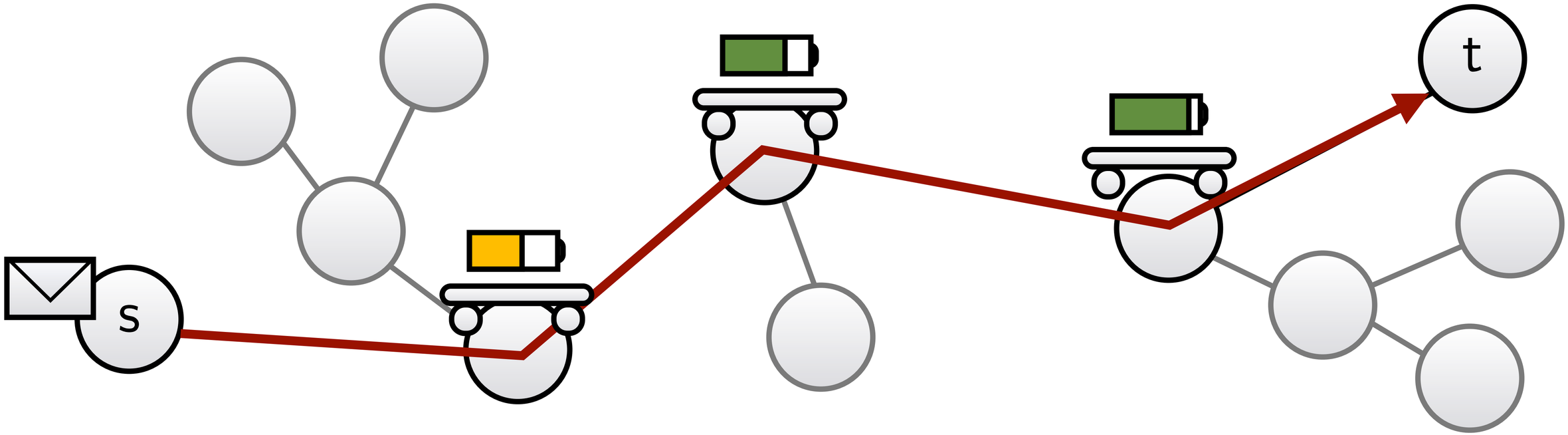


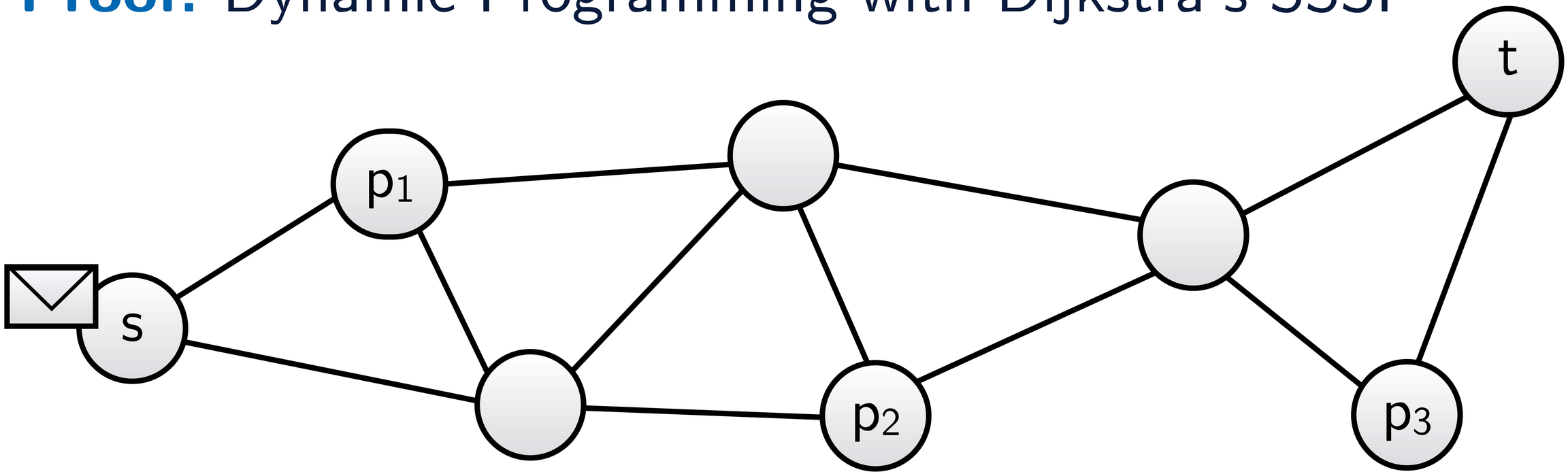
**Theorem:** There is a $O(n \cdot k + k \cdot \log k)$ algorithm for trees.




Surprising as non-returning setting is weakly NP-hard.
[Chalopin, Jacob, Mihalák, Widmayer, 2014]

**Theorem:** If agents are restricted to carry the message in a fixed order, we can decide in time $O(k(n+m)(n \cdot \log n + m))$.

**Proof:** Dynamic Programming with Dijkstra's SSSP



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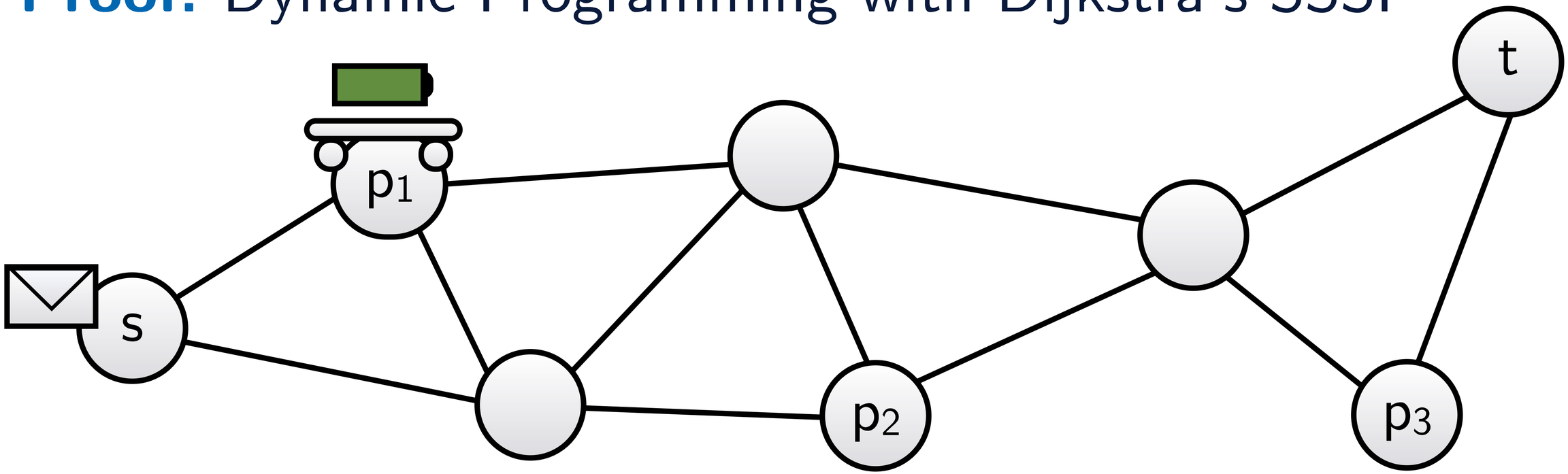

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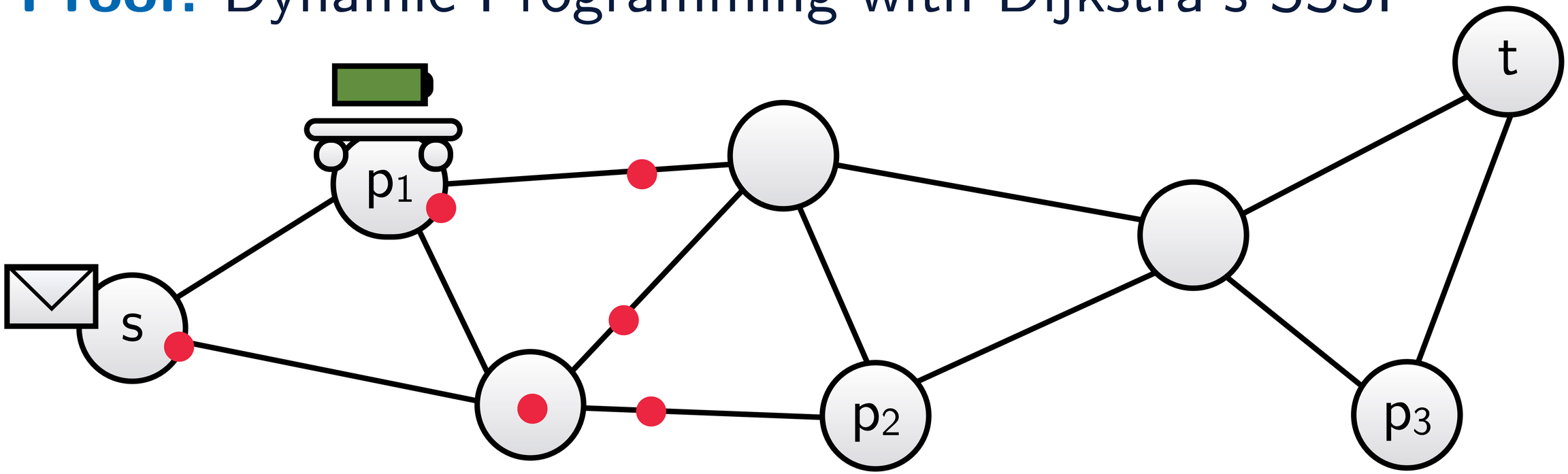

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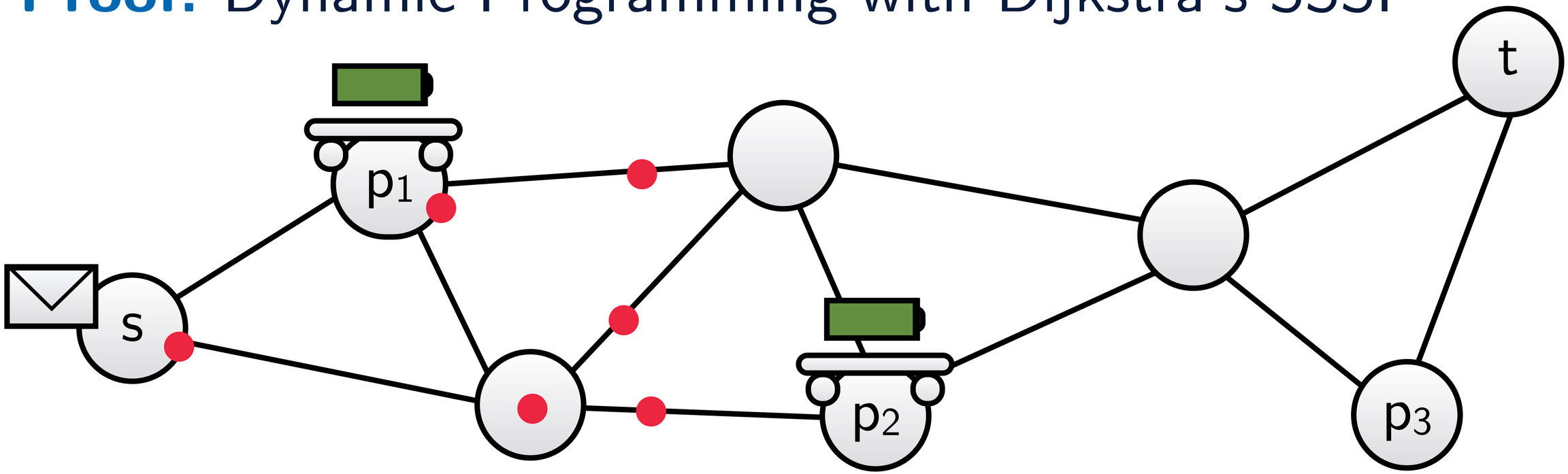

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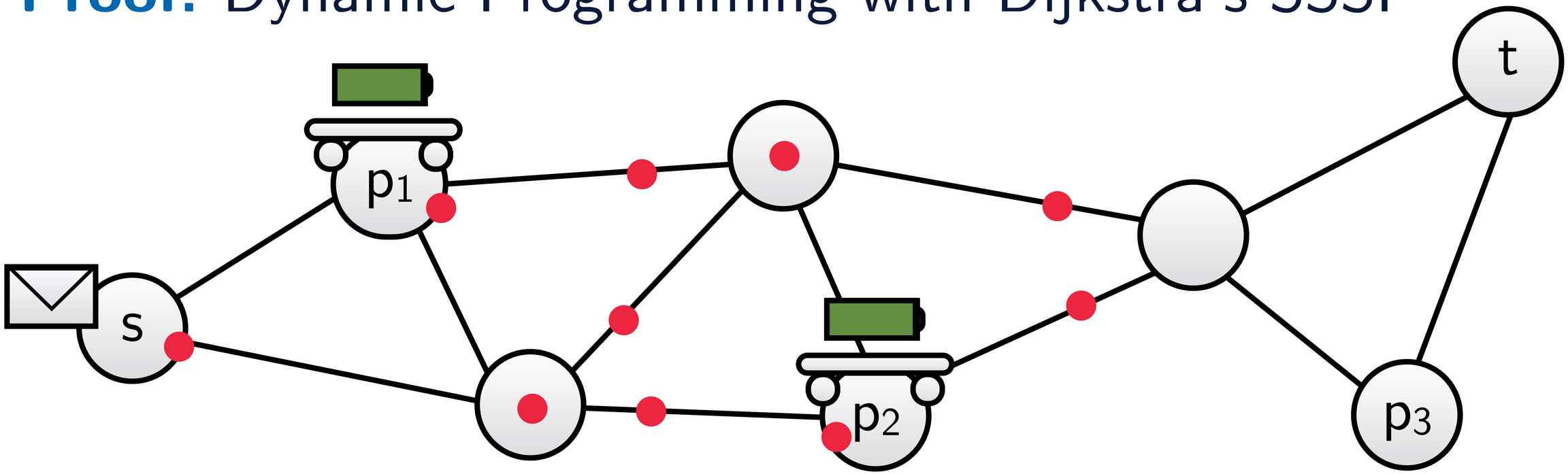

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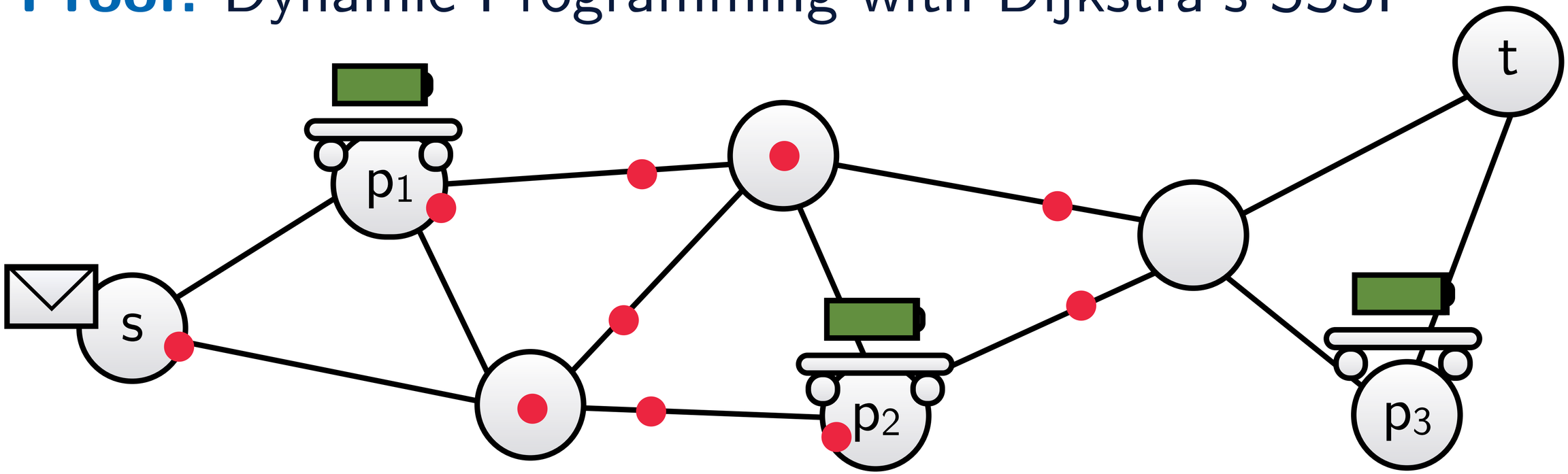

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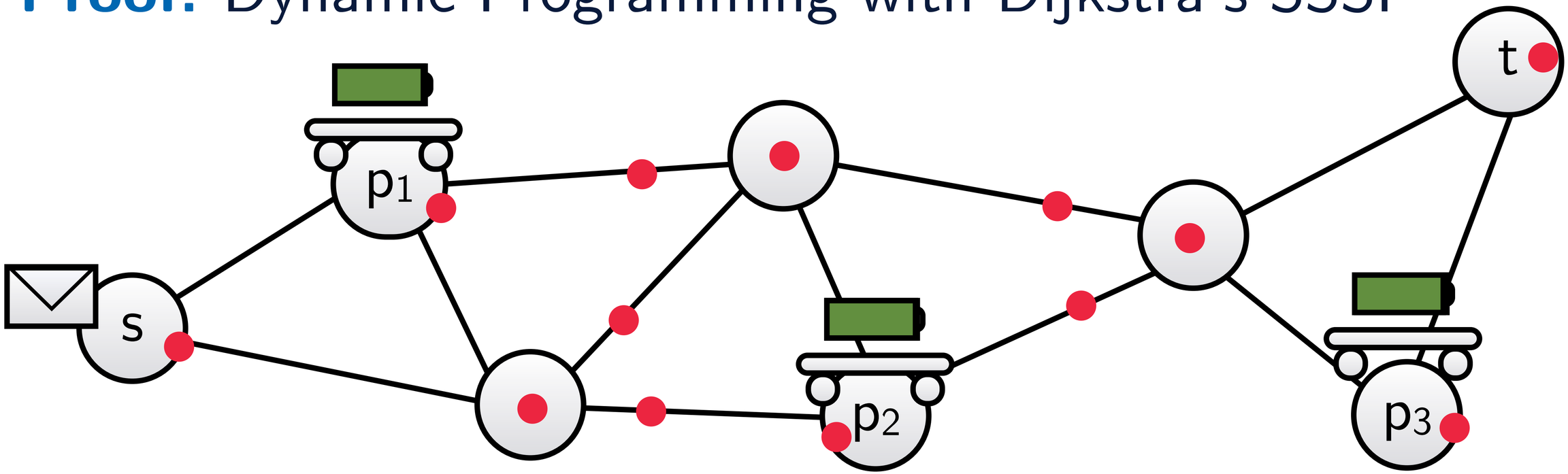

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**Corollary:** For constantly many agents, we can do it in poly(n,m) by brute forcing the order of the agents.

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**Corollary:** For constantly many agents, we can do it in poly(n,m) by brute forcing the order of the agents.

**Question:** What about many agents?

**Theorem:** Our problem is strongly NP-hard even on planar graphs.

**Proof Sketch:** Reduction from Planar-3SAT.

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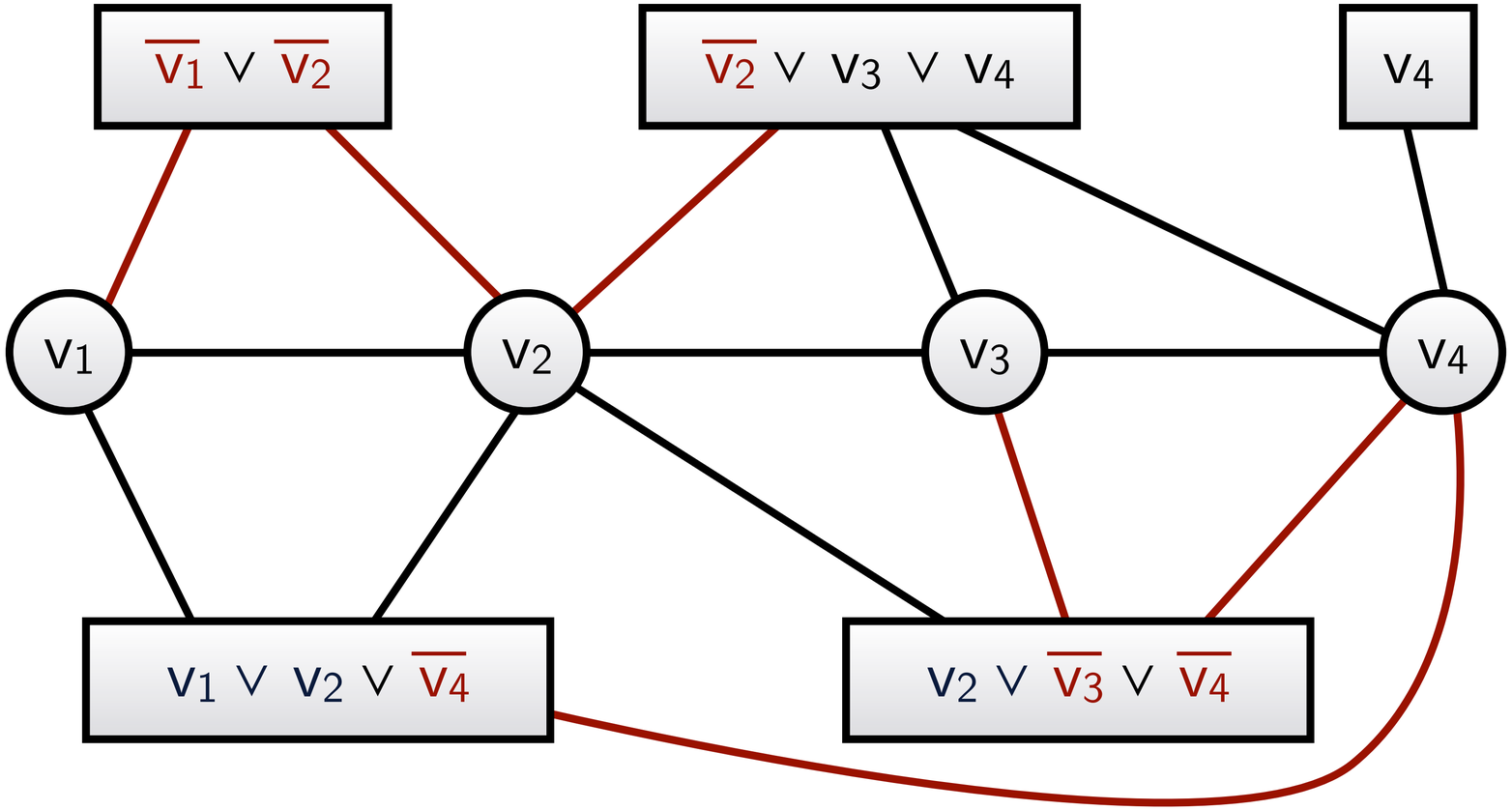

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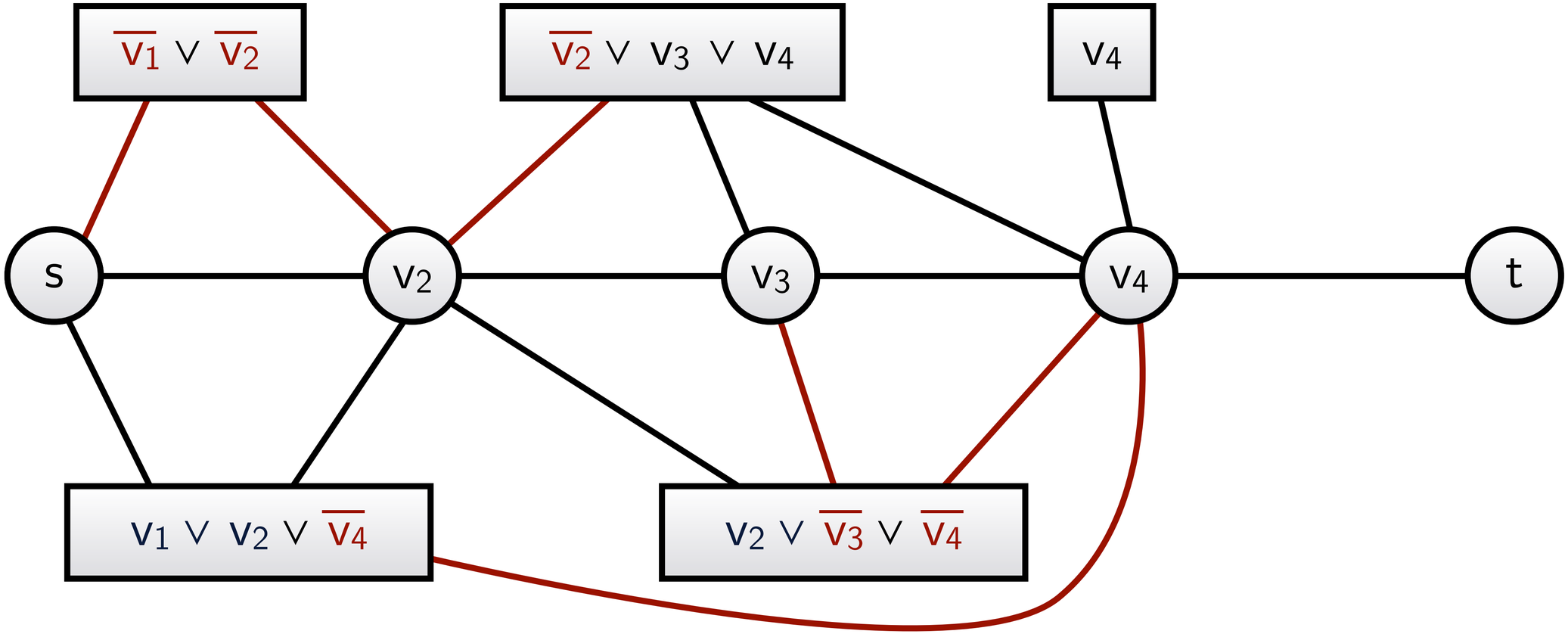

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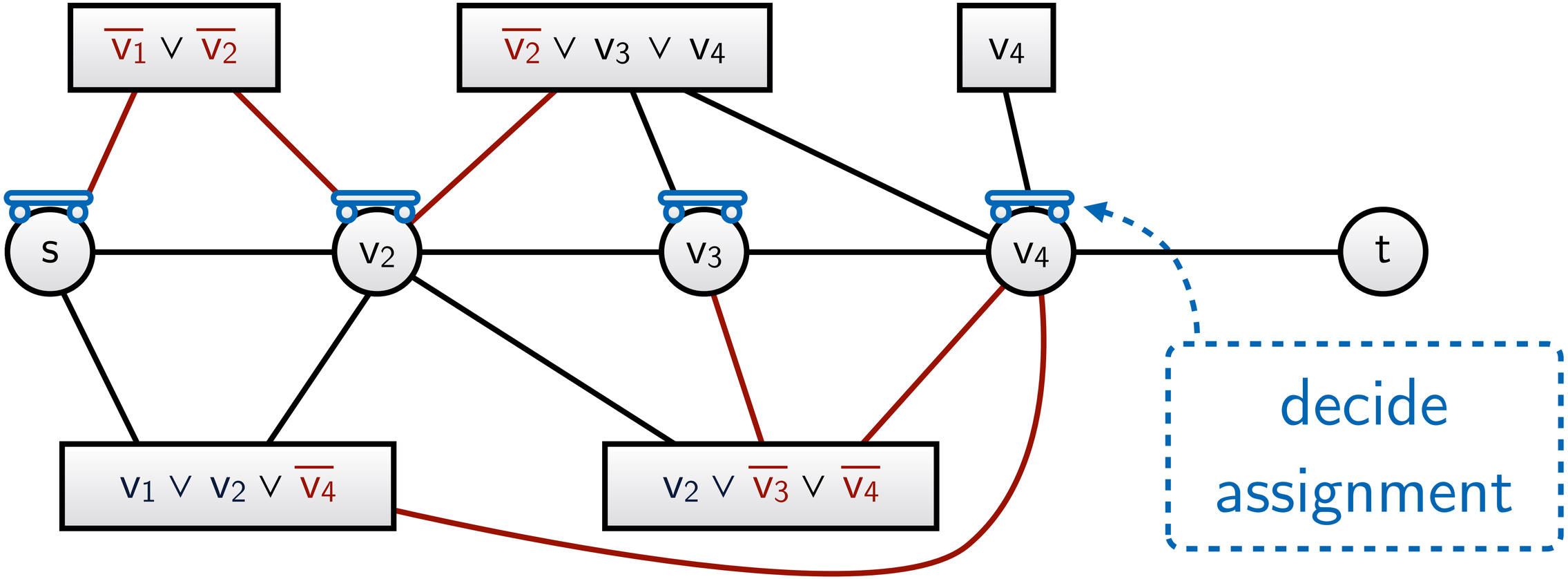

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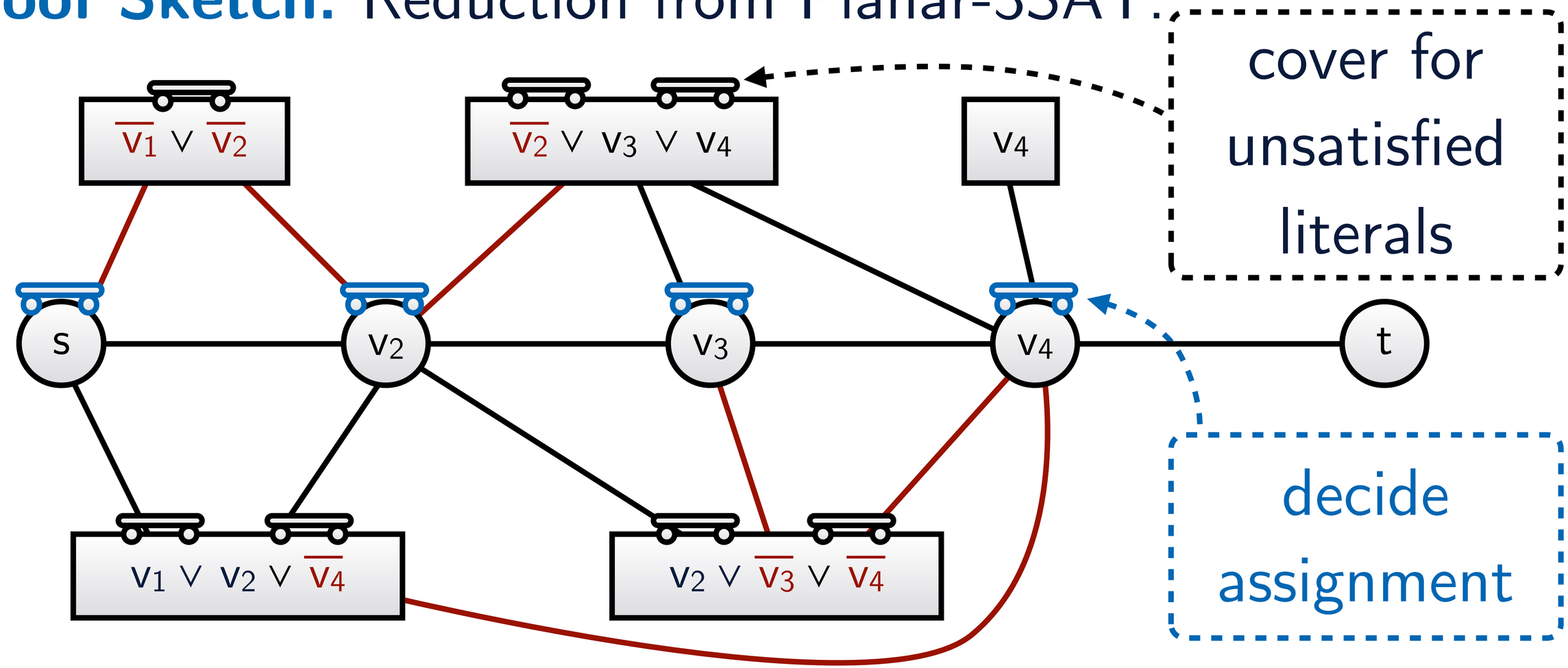

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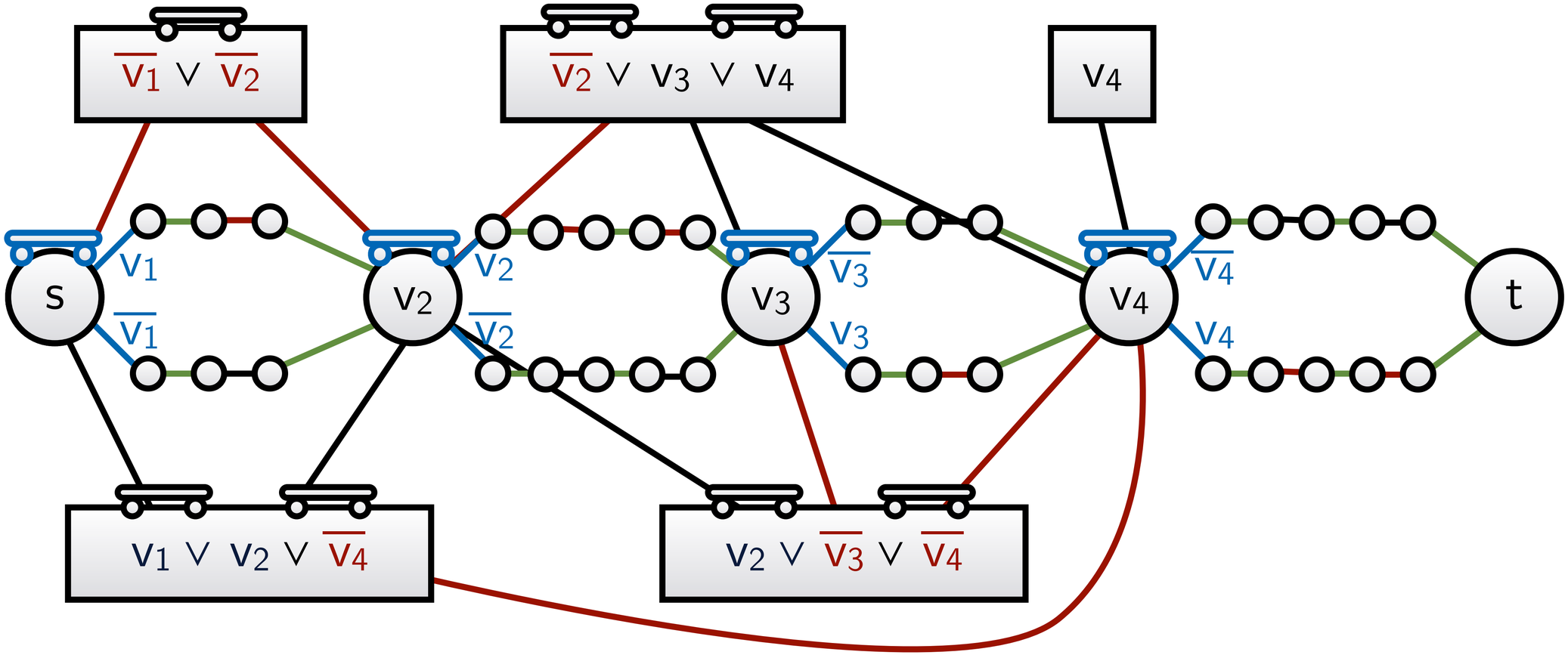

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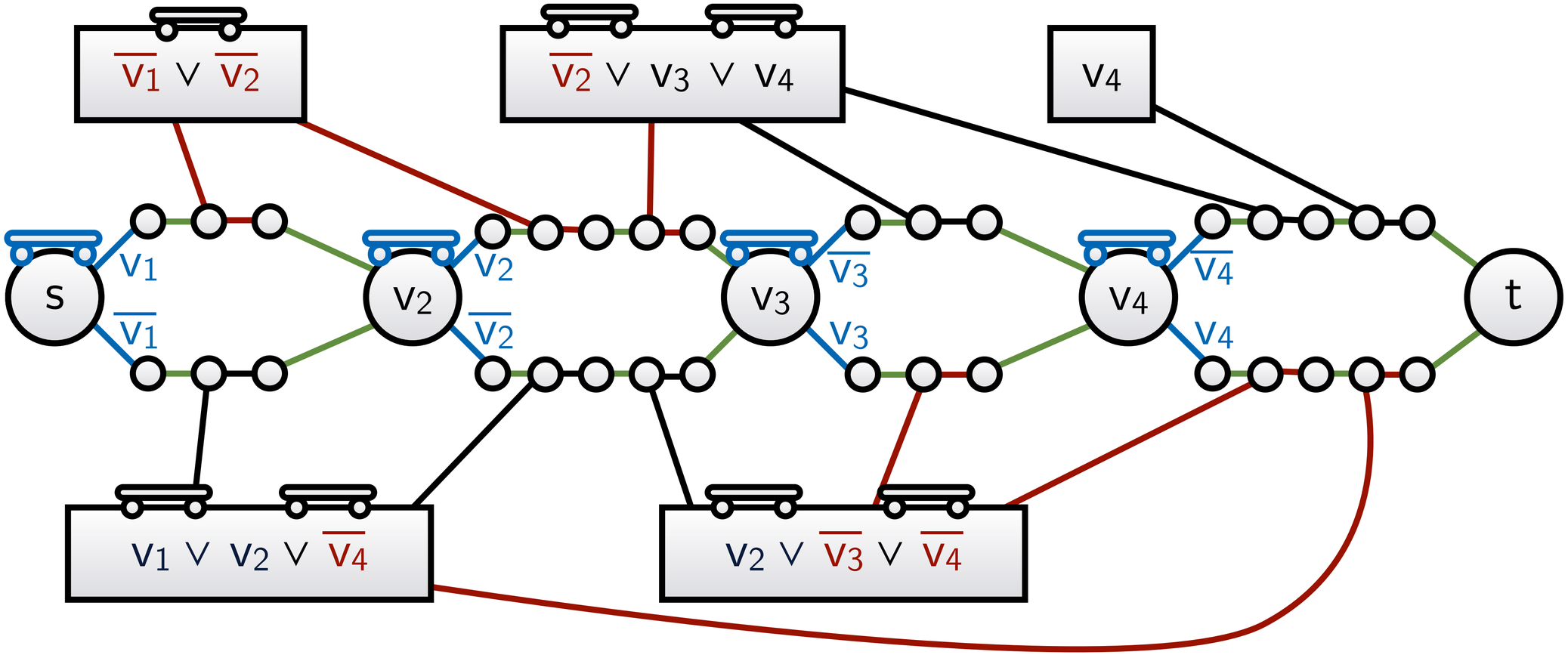

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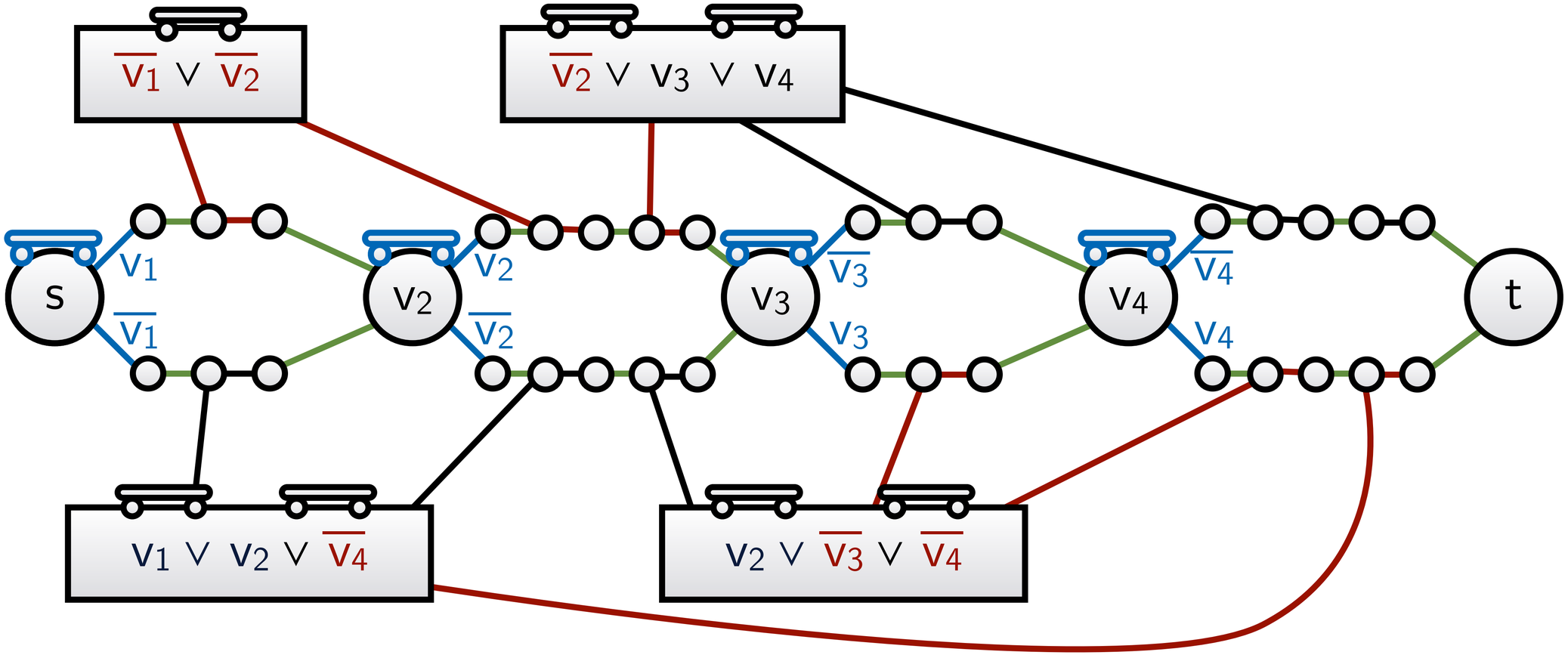

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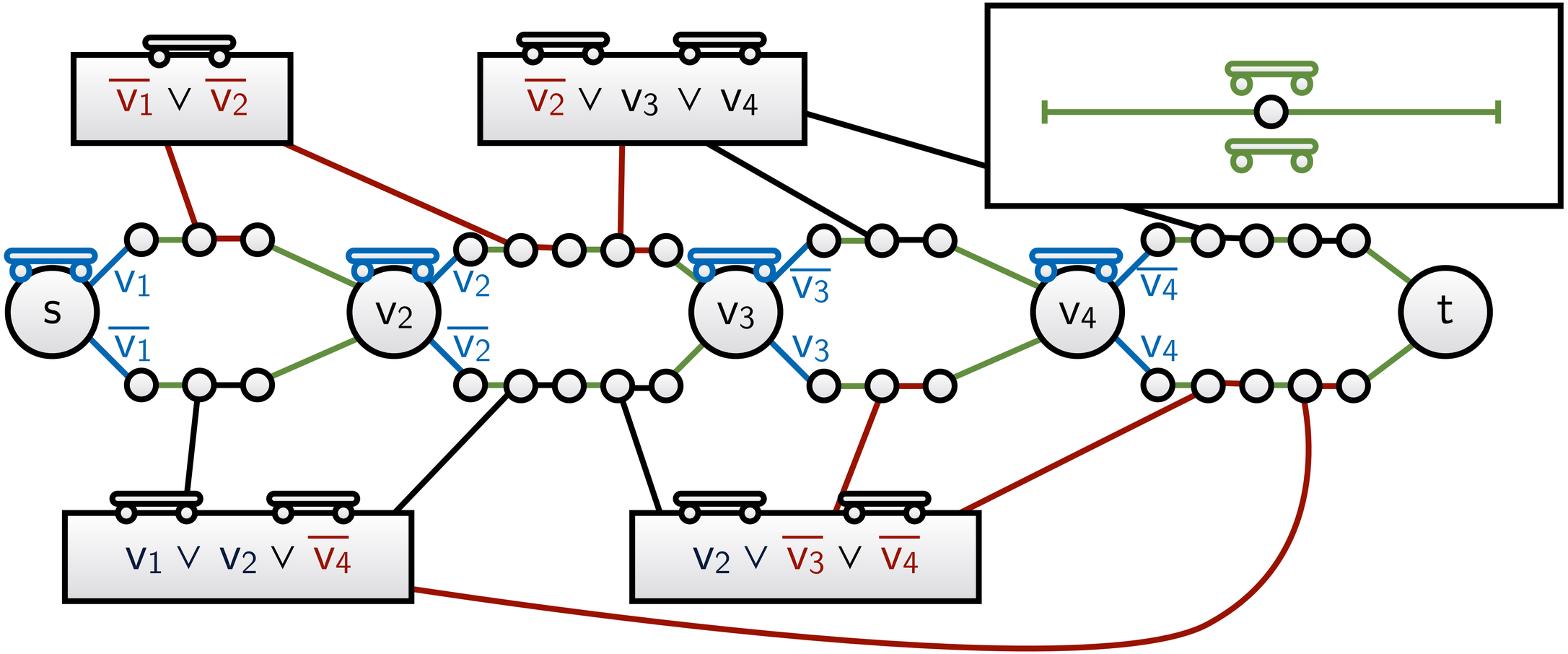

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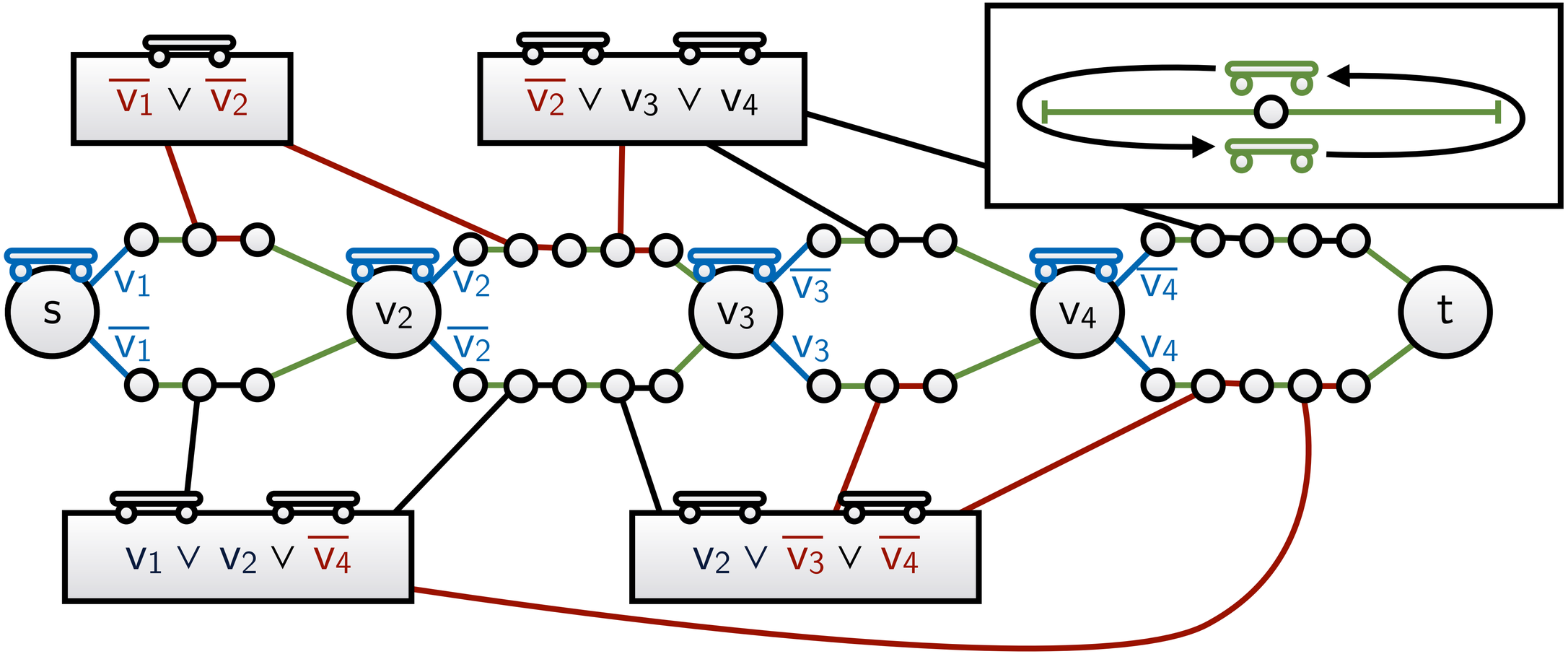

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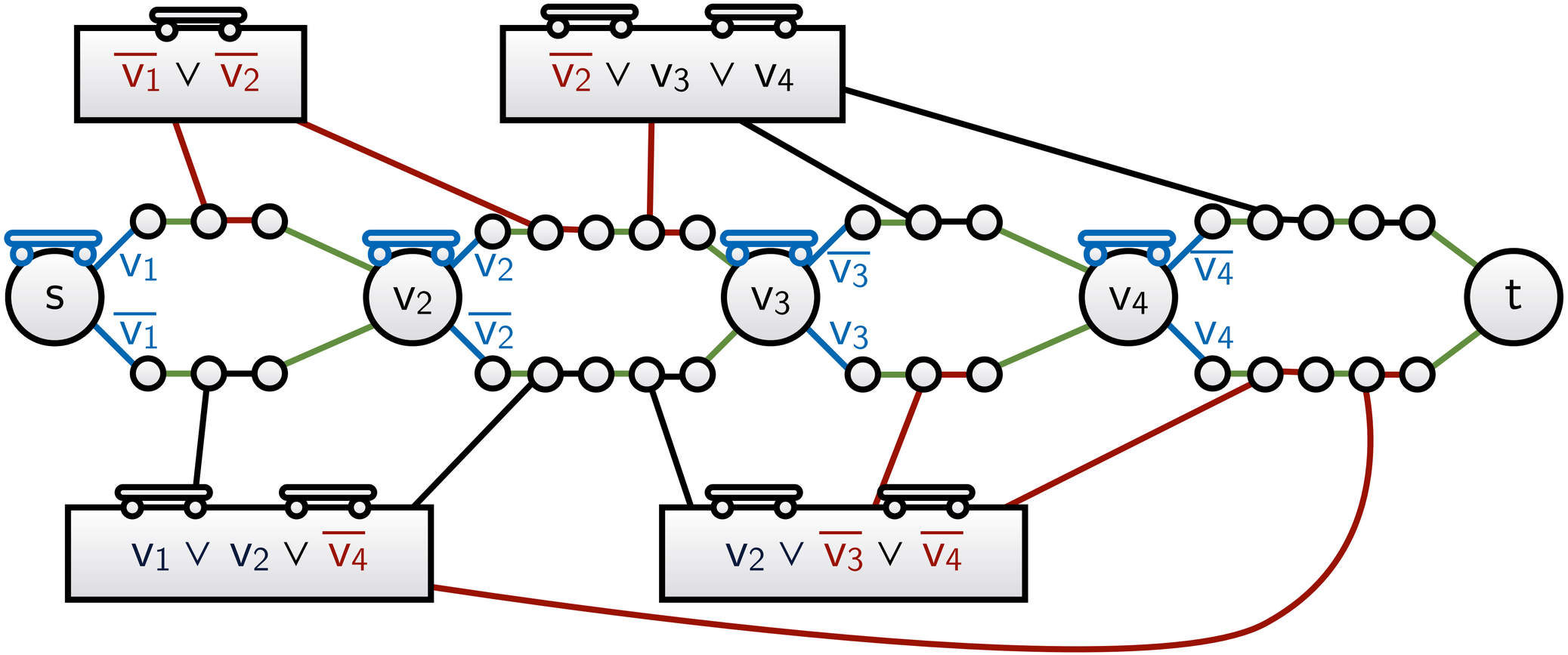

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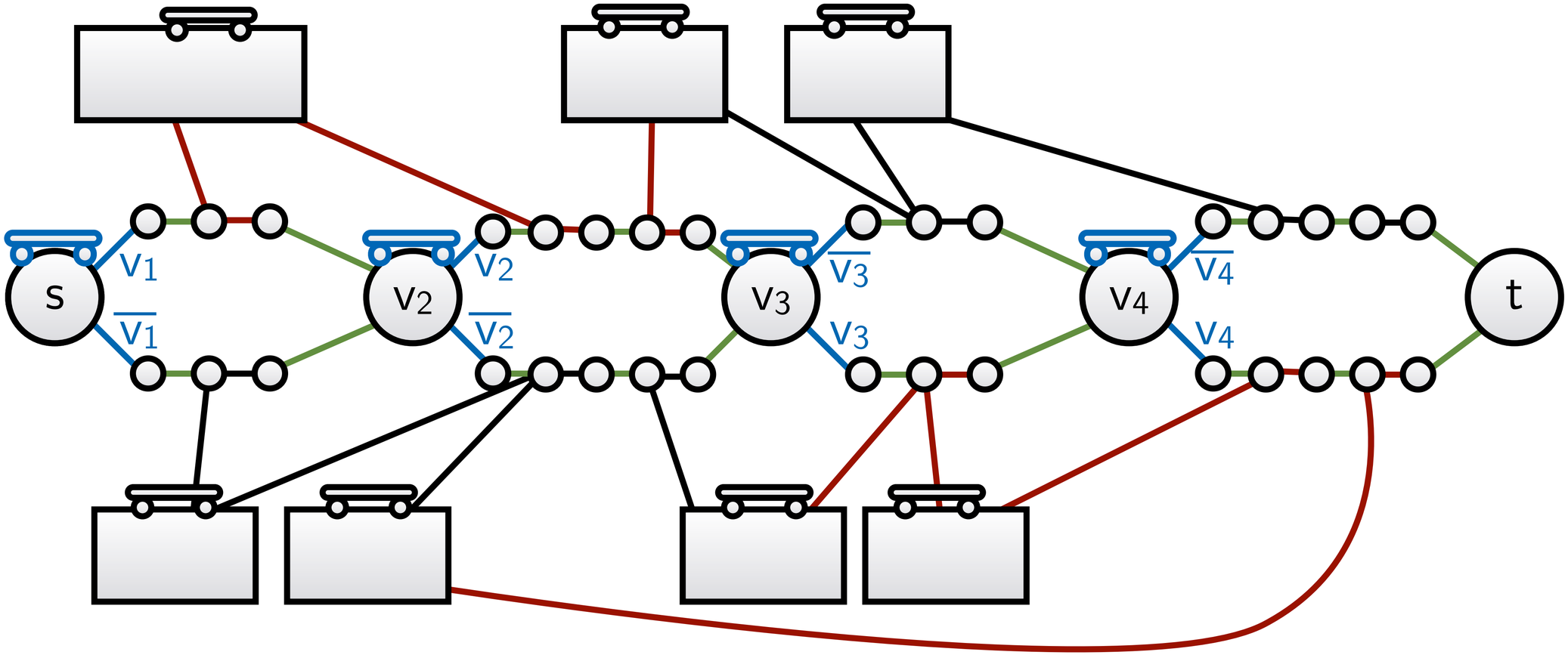

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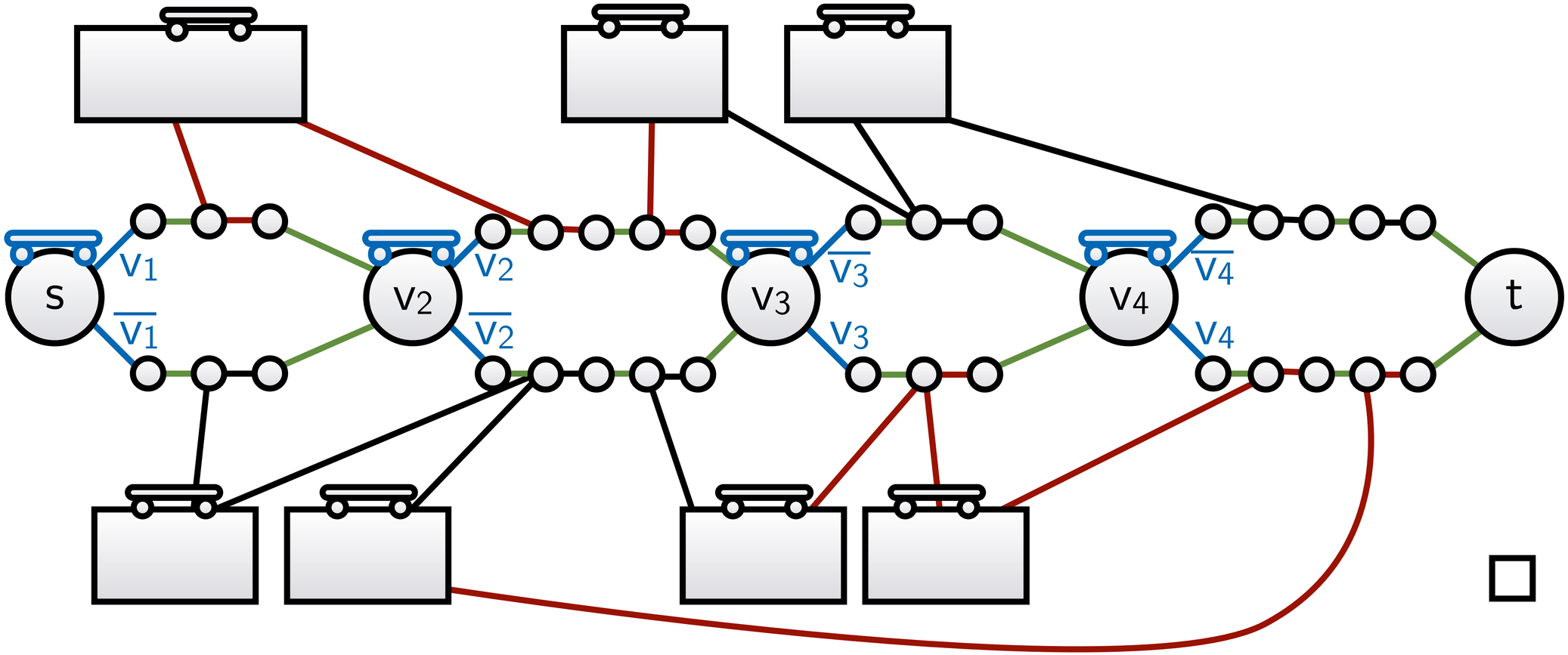


□

# Resource Augmentation / Definition

What is a 2-resource-augmented algorithm?

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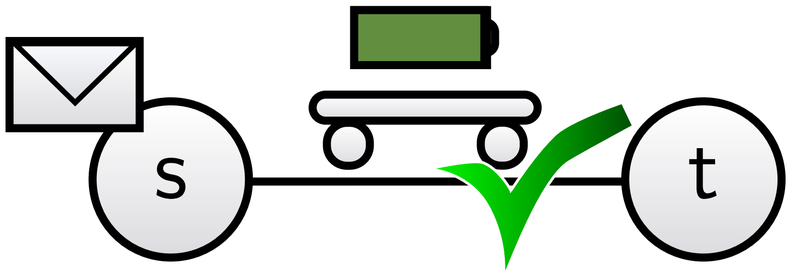

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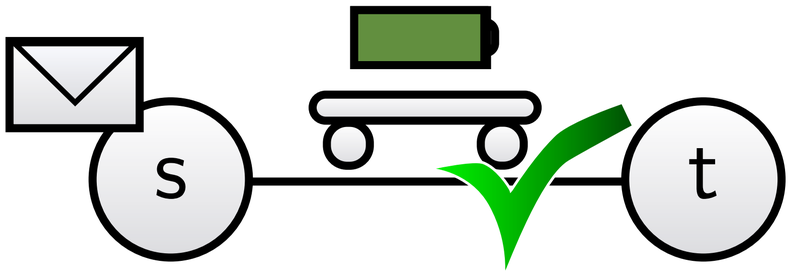


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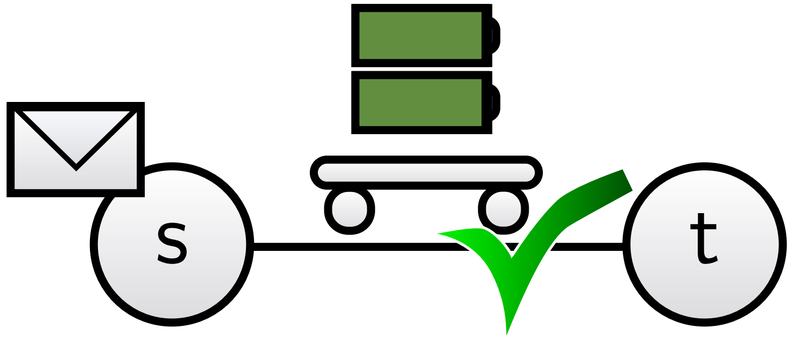

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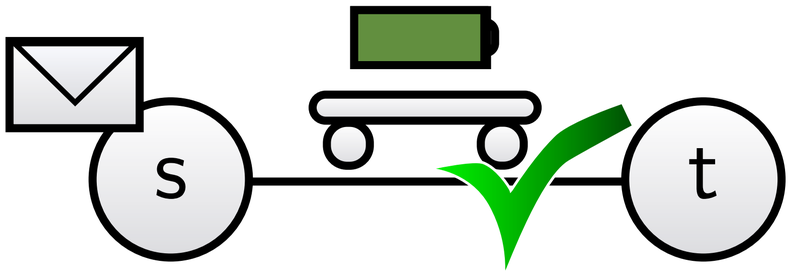


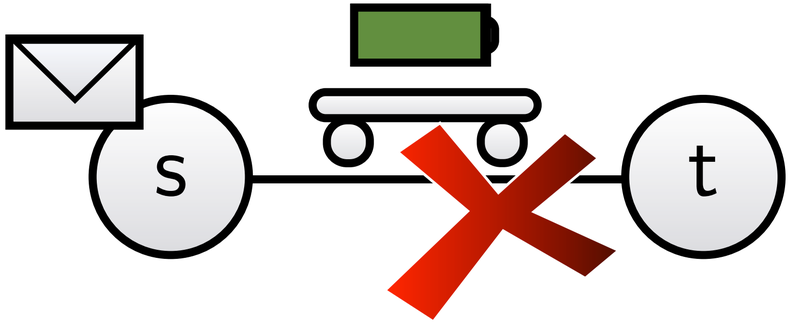


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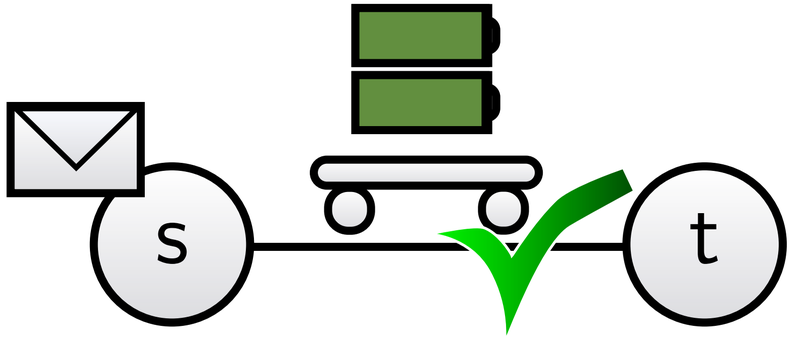

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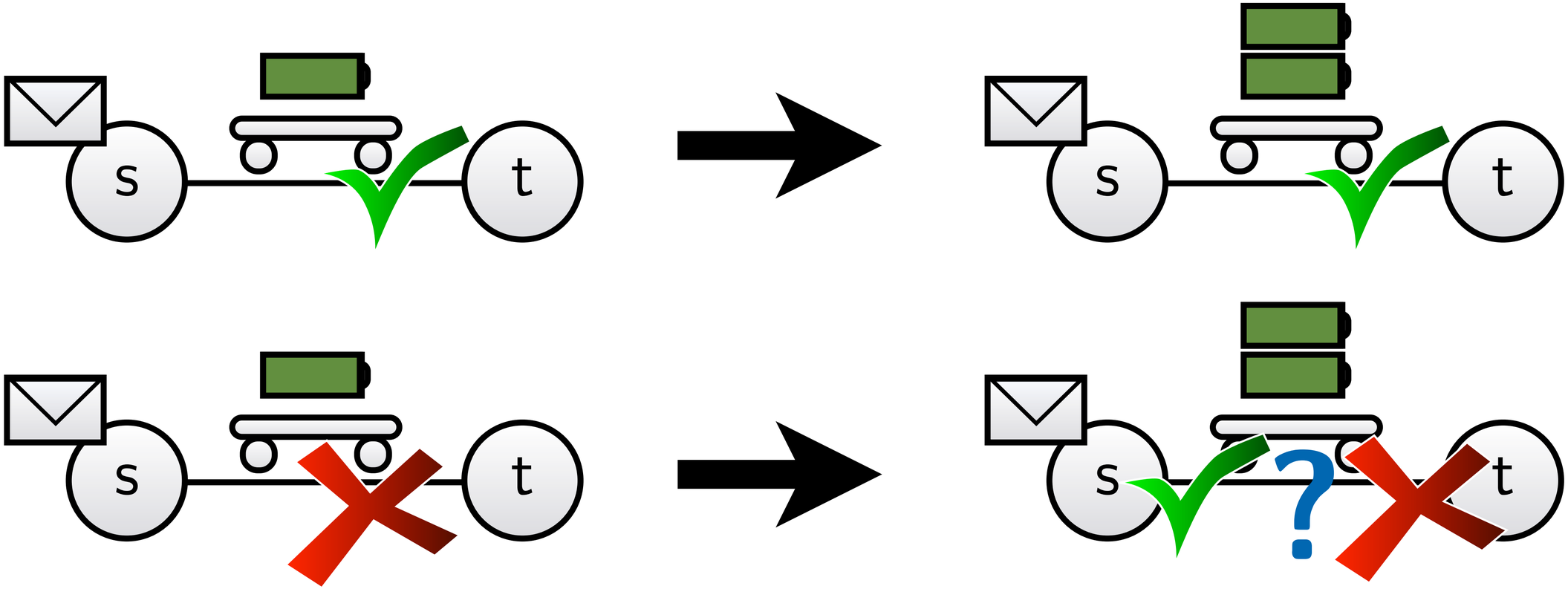
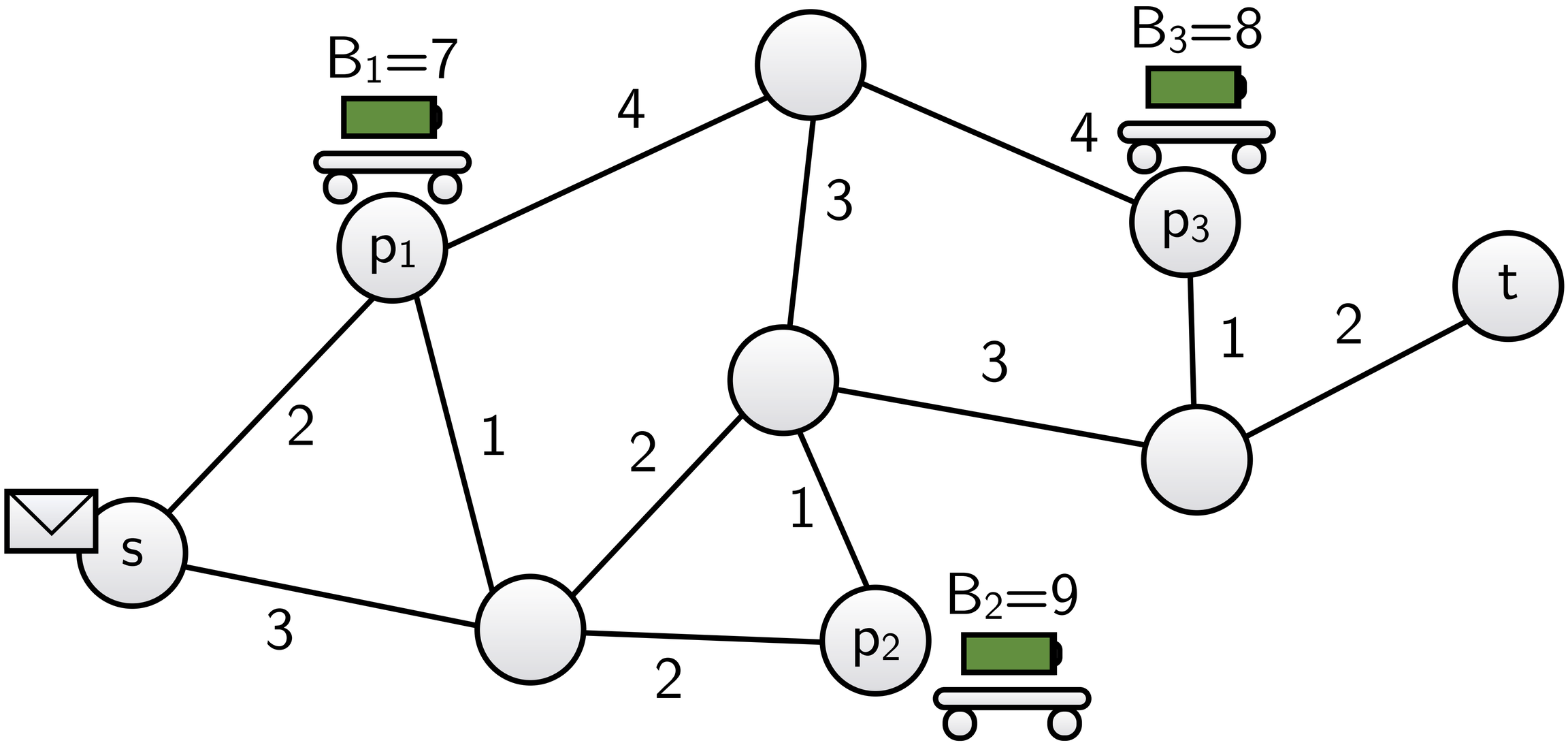


# Resource Augmentation / Example

ETH zürich



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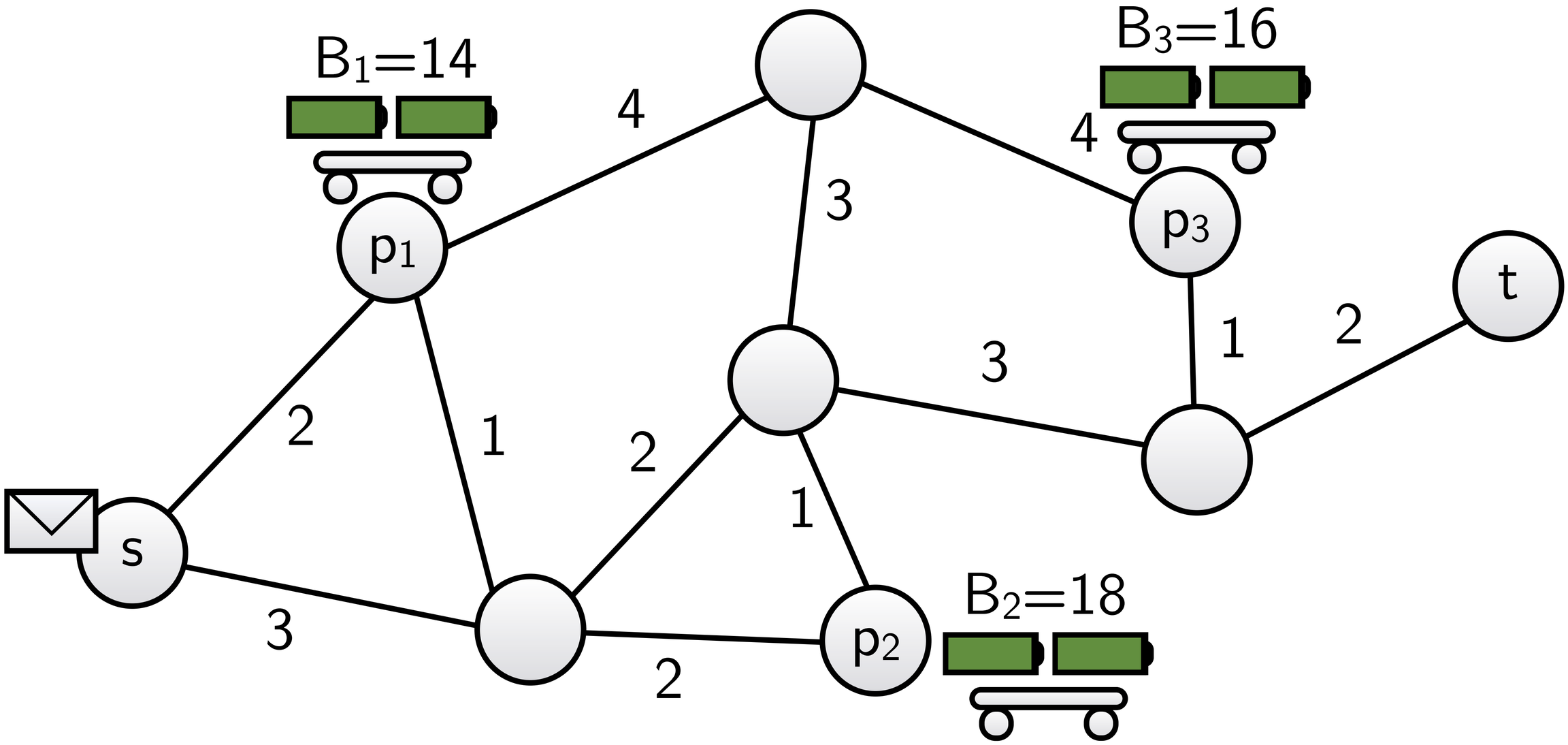

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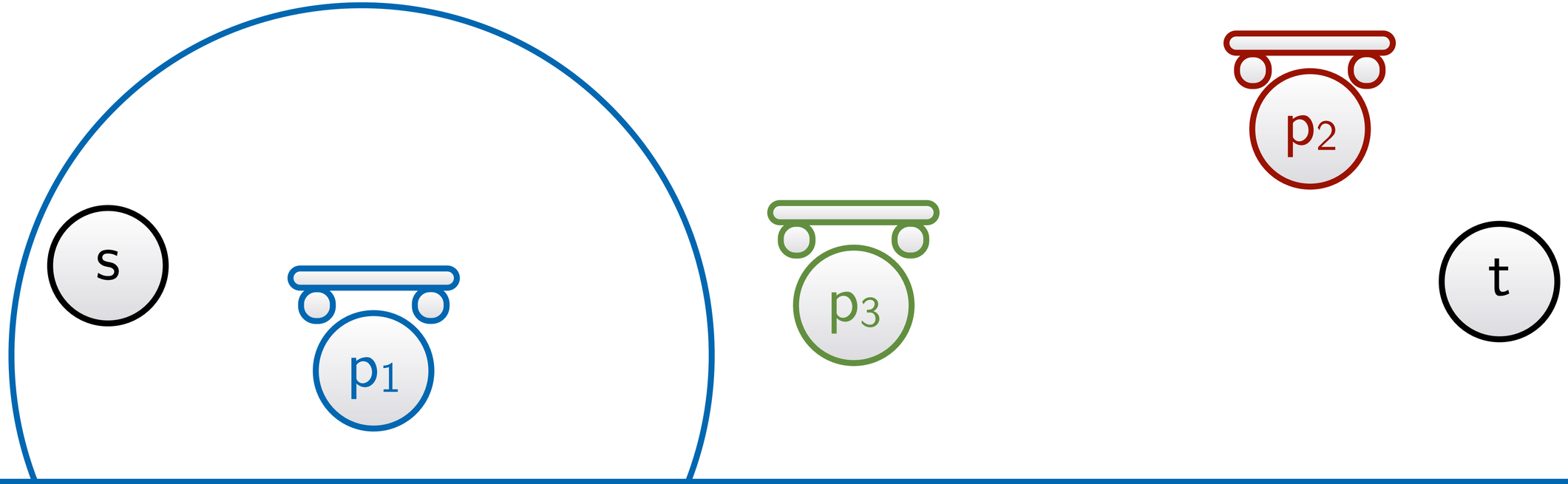

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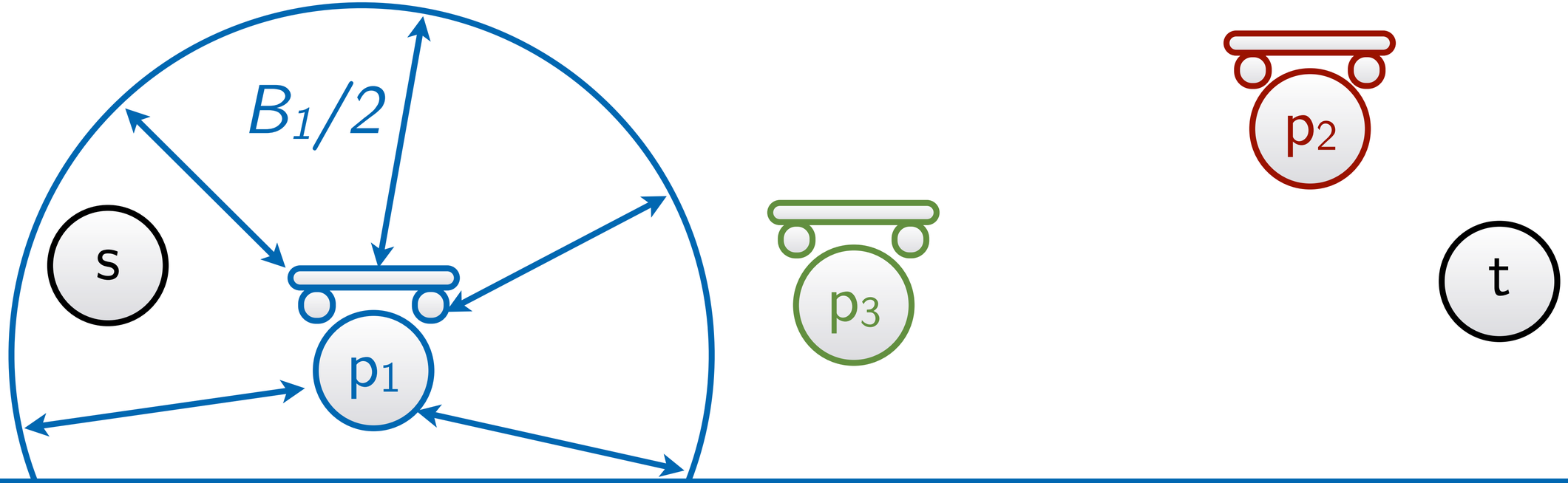

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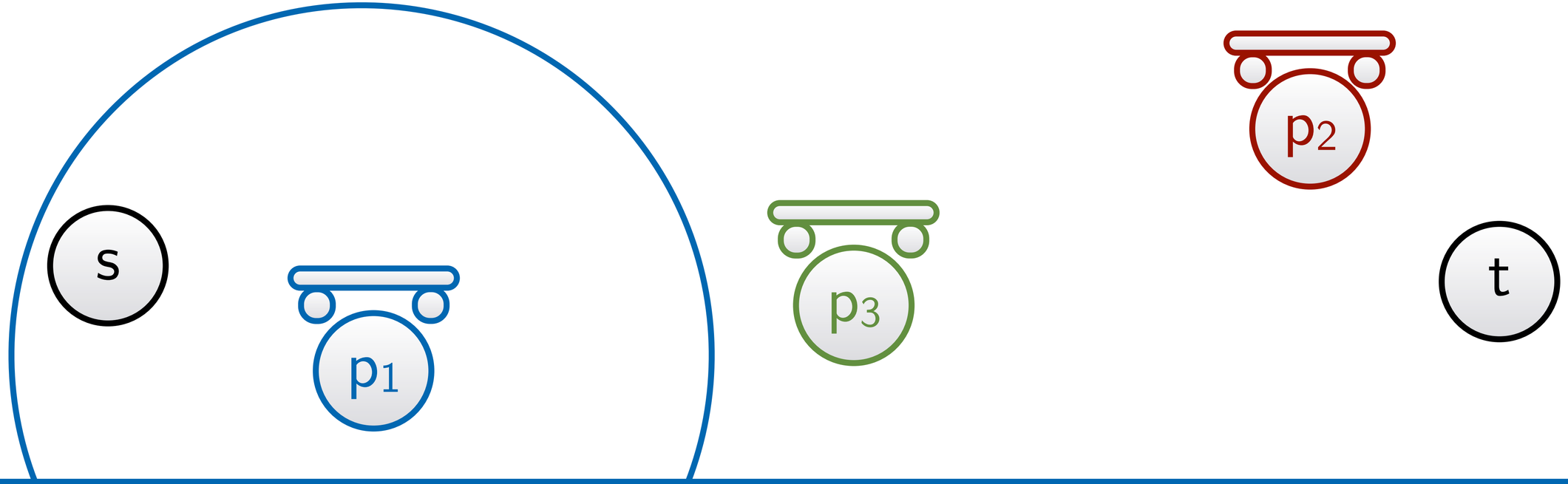

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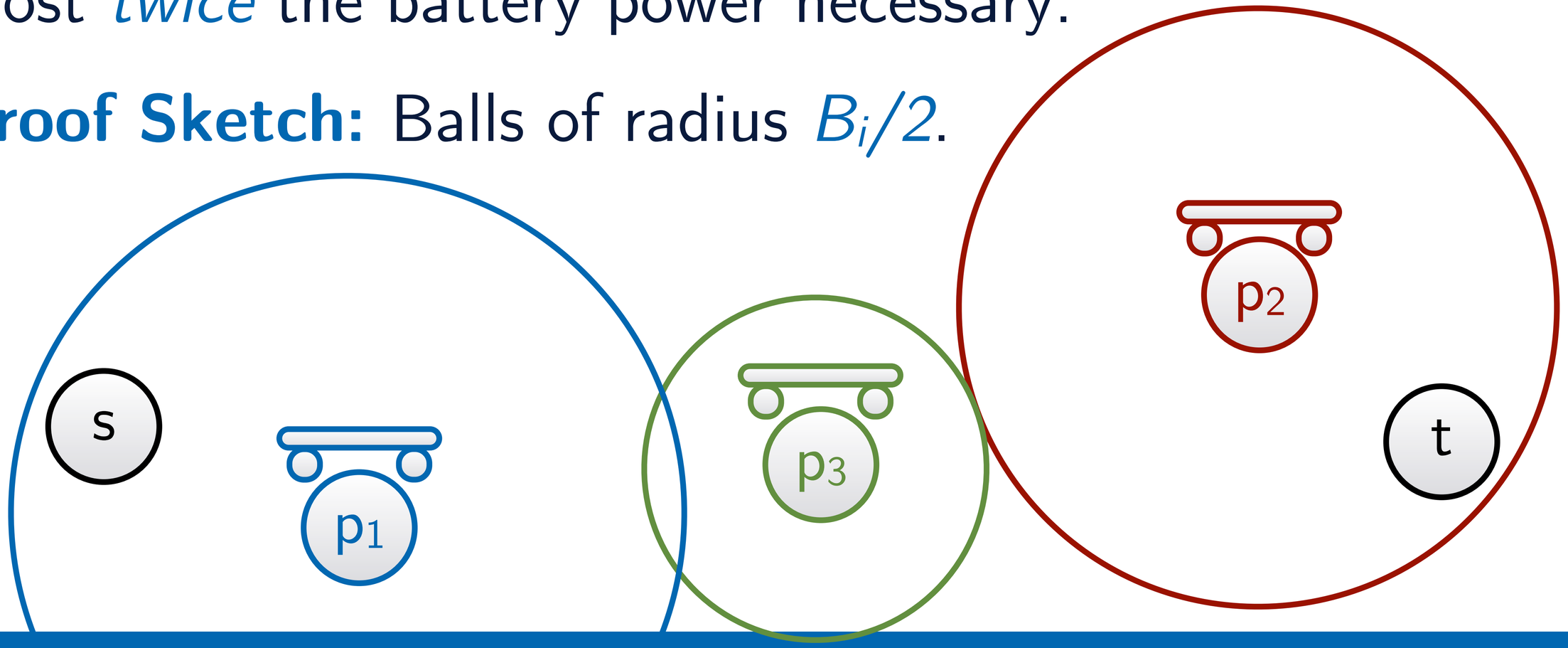

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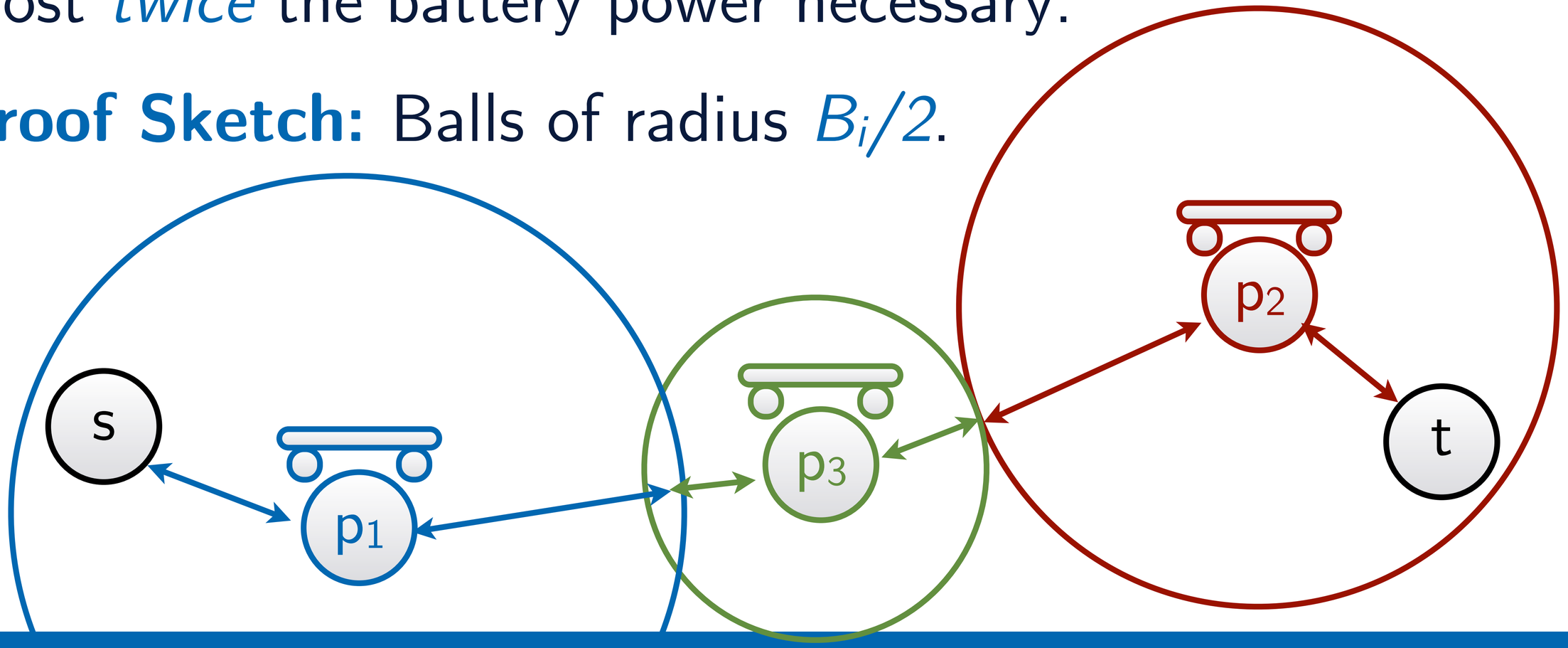

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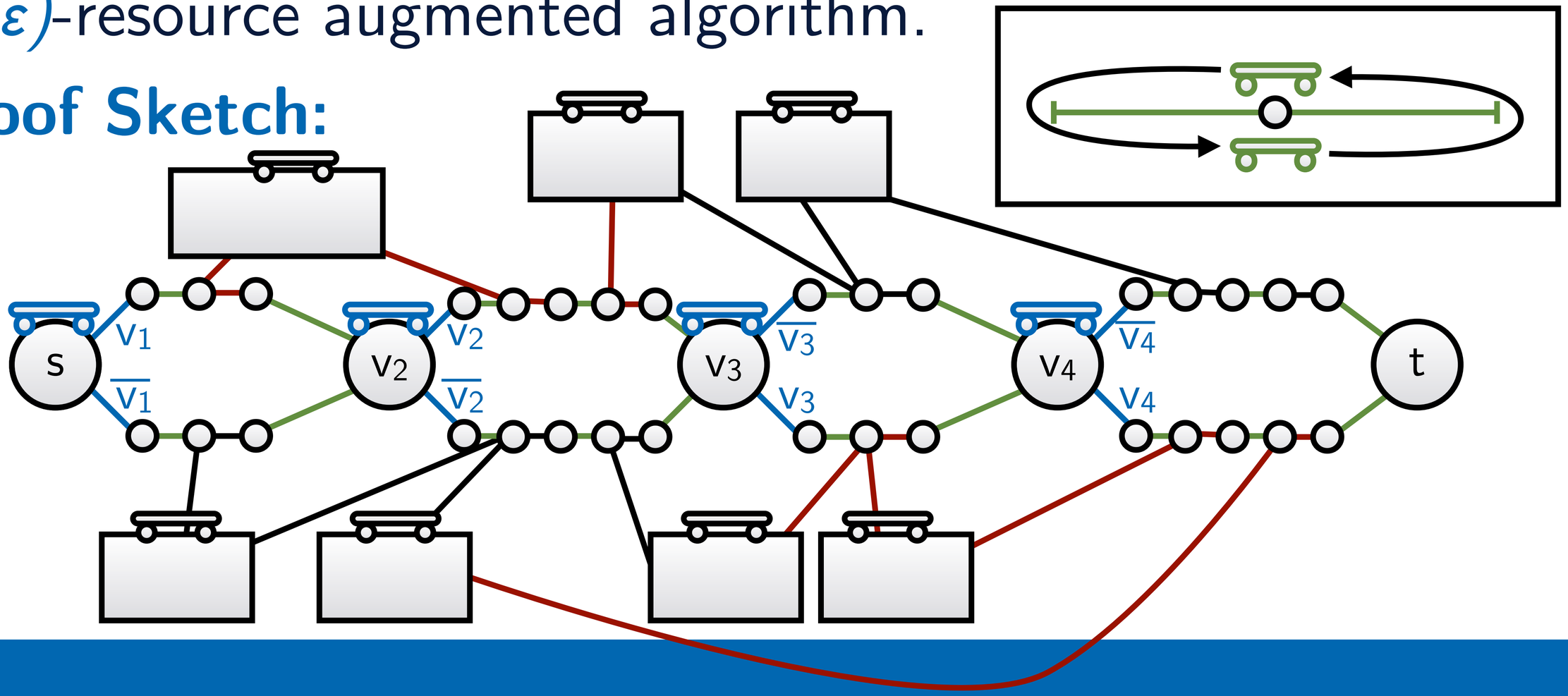

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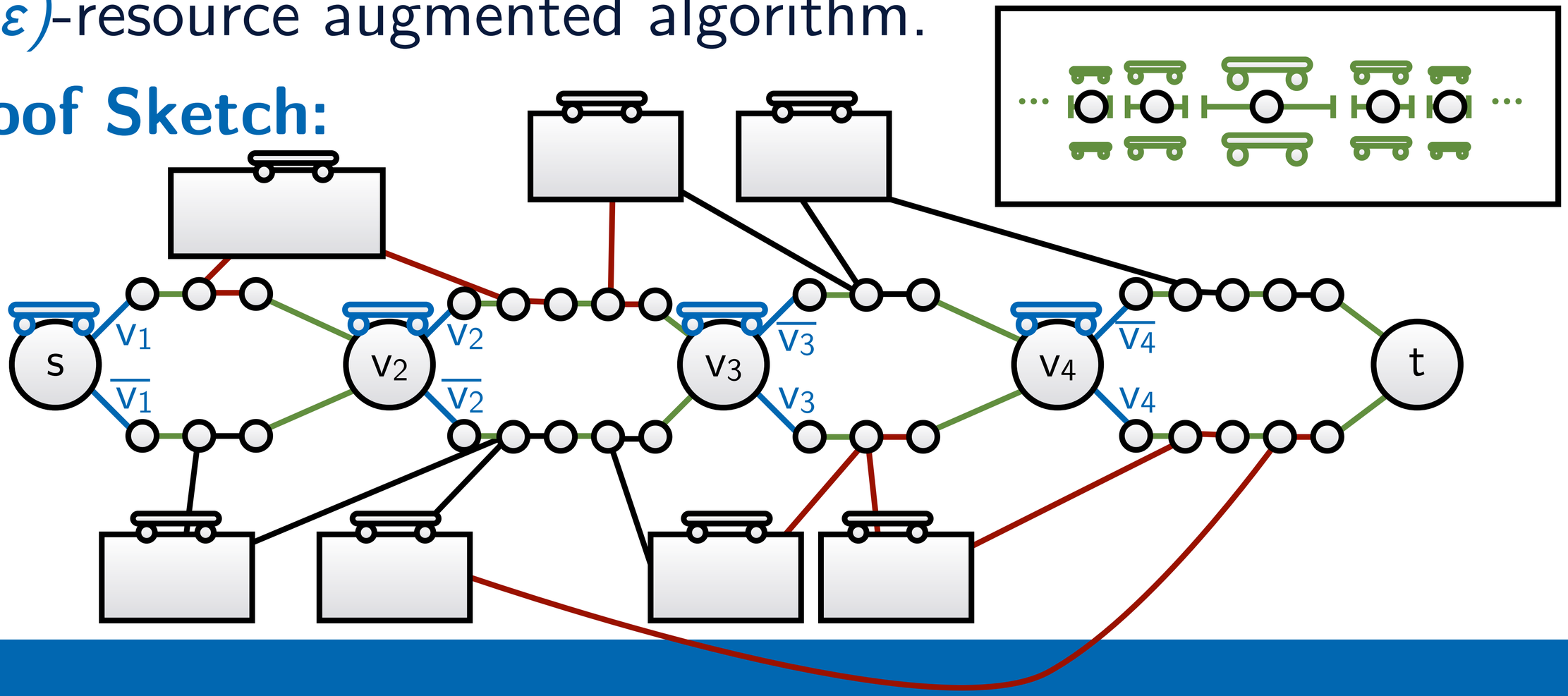

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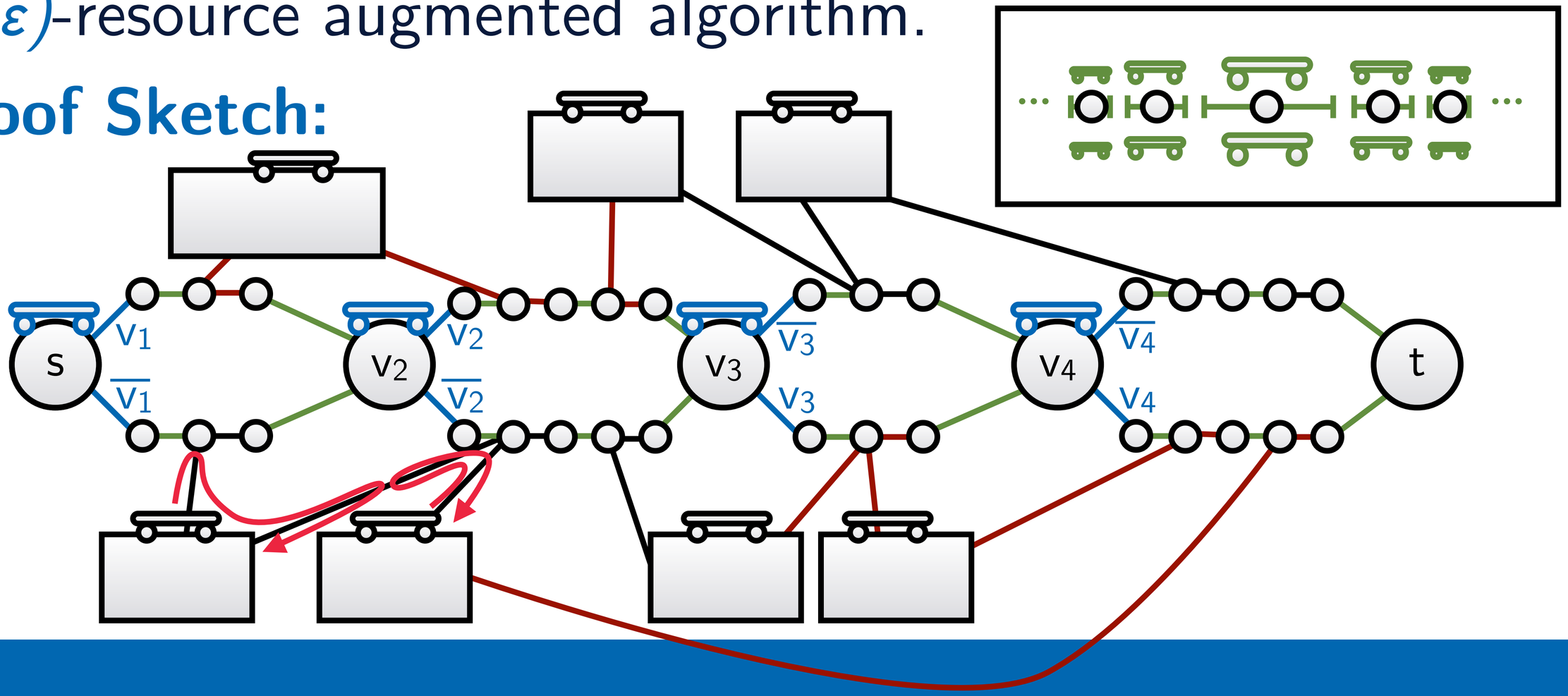

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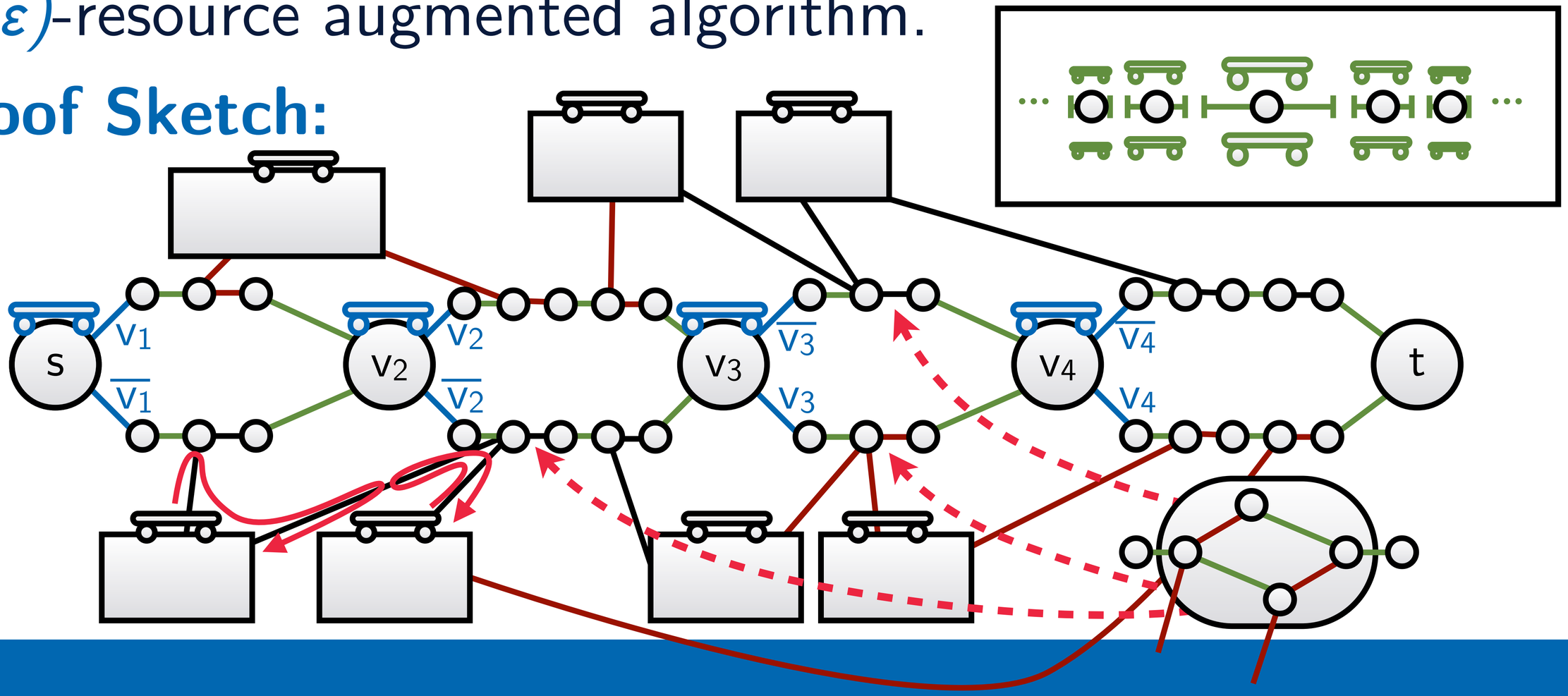

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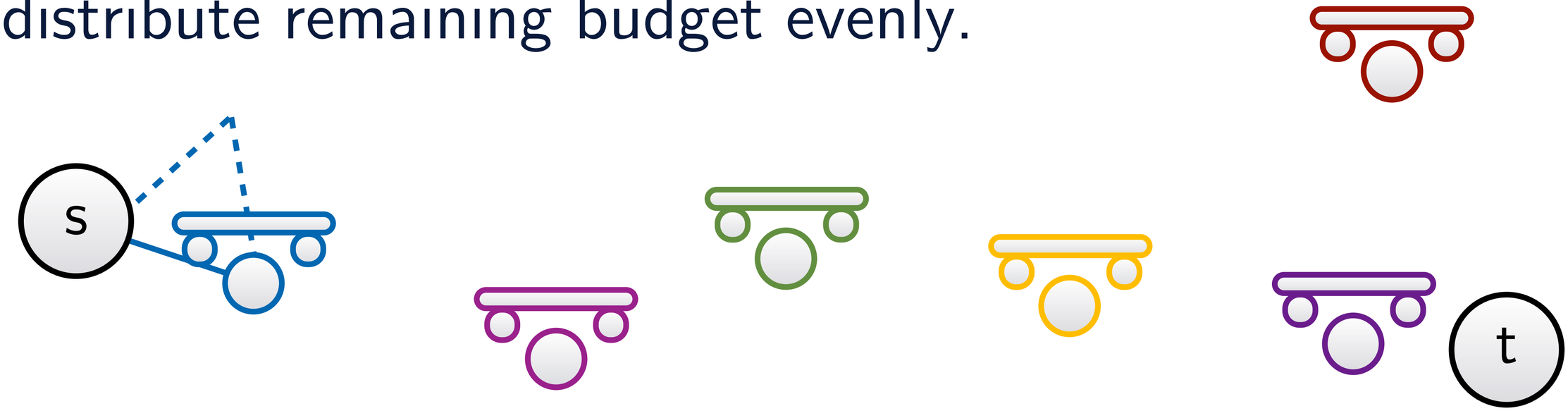

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## Contributions

- fast, greedy algorithm for paths and trees
- strong NP-hardness for planar graphs
- 2-resource-augmentation algorithm and it is tight
- more for non-returning case

## Future Work

- multiple messages
  - on a line, or
  - with fixed agent order

**Thank you!**